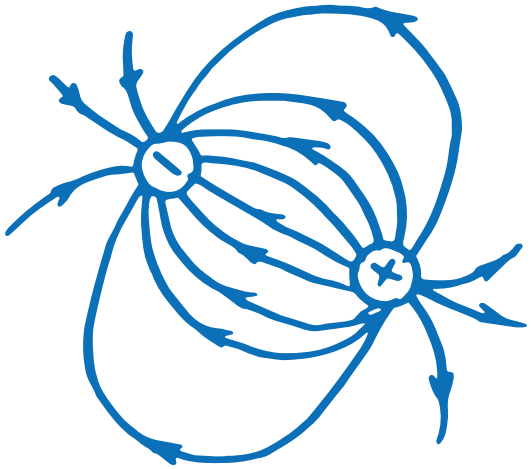
PROBLEMS AND SOLUTIONS 17

UNIVERSITY PHYSICS

ELECTRIC CHARGE AND ELECTRIC FIELD

100+ PROBLEMS & SOLUTIONS

PHONGSAK CHINNABOON



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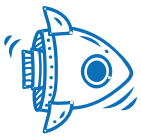
About the Author

Phongsak Chinnaboon received his bachelor's degree in engineering from Chulalongkorn University, Thailand. Over the past 10 years, he has authored more than 20 physics books at high school and university levels. He has also taught physics on Youtube, which is one of the most popular channels with over 1000 physics videos.

His books and lectures are well known for more detailed, step-by-step, and easy-to-understand content. Moreover, he has great power in illustrating more than 100 examples for each lesson to show all aspects of physics theories' application. Many students have achieved higher scores, good grades, and passed examinations to their desired universities.



Problems and Solutions in University Physics

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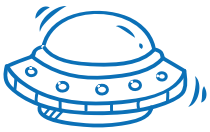


Preface

Most students encounter this trouble when studying physics:

"They study physics in their classroom but could not do any problem, even a basic one."

On top of that, when they cannot solve one lesson's problems, the next lesson's problems are always more complicated. Finally,

"Physics has become a complicated subject."

The solution to this trouble is,

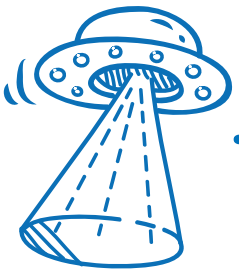
"Learn more deeply by doing more problems."

Enough problems covering various aspects of physics theories will help students understand physics better and improve problem-solving skills.

This book has the following highlights:

- 100+ problems, from basic to calculus-based ones. This makes students very expert in applying physics theories.
- Detailed and step-by-step solutions to every problem. This allows students to practice problem-solving skills by themselves.
- At the end of this book, there are blank lined pages for taking notes of what students have learned from doing the problems.

Phongsak Chinnaboon



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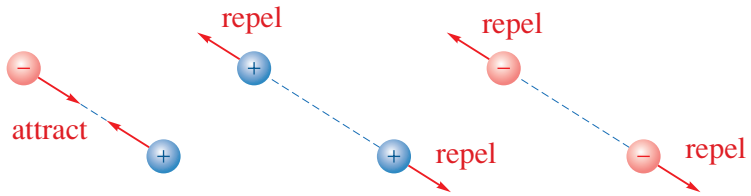
Summary

Electric Charges have the following properties:

- Charges of opposite signs attract each other; on the other hand, charges with the same sign repel each other. Alternately, briefly, "Opposite charges attract, and like charges repel."



- The conservation law of electric charges states that "a total electric charge (algebraic sum of all positive and negative charges) of a closed system is always constant.
- Electric charge is quantized, and the smallest charge is the charge of an electron or proton that has a magnitude of $e = 1.60 \times 10^{-19}$ C.

Conductors are materials that electrons can move freely; most metals are good conductors.

Insulators are materials that electrons cannot move freely; most nonmetals are insulators.

Coulomb's law: Charges q_1 and q_2 separated by a distance r exert electric forces on each other; the magnitude of the force is proportional to the product q_1q_2 and inversely proportional to r^2, as follows:

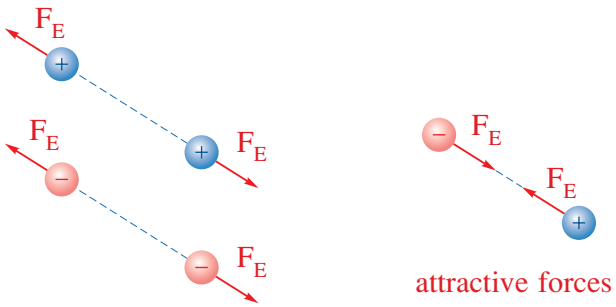
$$F_E = k_0 \frac{|q_1 q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{|q_1 q_2|}{r^2}$$

where $k_0 = \dfrac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$.

k_0 is called "Coulomb constant."

ε_0 is called "the permittivity of free space" and equal to $8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$.

- The electric force on each charge is along the line joining the two charges q_1 and q_2. If both charges have the same sign, the electric force is repulsive; on the other hand, the force attractive when having opposite signs.



- If two or more charges exert electric forces on another charge, the resultant electric force on that charge is the vector sum of the forces due to each charge.
- An electric force is written in vector form as

$$\vec{F}_{12} = k_0 \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

where $\hat{r}_{12} = \dfrac{\vec{r}_{12}}{r_{12}}$.

$\hat{r}_{12}$ is a unit vector to locate charge q_1 considered by charge q_2, pointing from q_2 to q_1.

$\vec{r}_{12}$ is the positioning vector of charge q_1 regarded by charge q_2.

r_{12} is the distance between charges q_1 and q_2.

Electric Field at any point in space is defined as electric force $\vec{F}_{E0}$ exerted on a small positive test charge q_0 at that point divided by the test charge q_0:

$$\vec{E} = \frac{\vec{F}_{E0}}{q_0}$$

- Electric field due to a point charge q at a distance r is given by

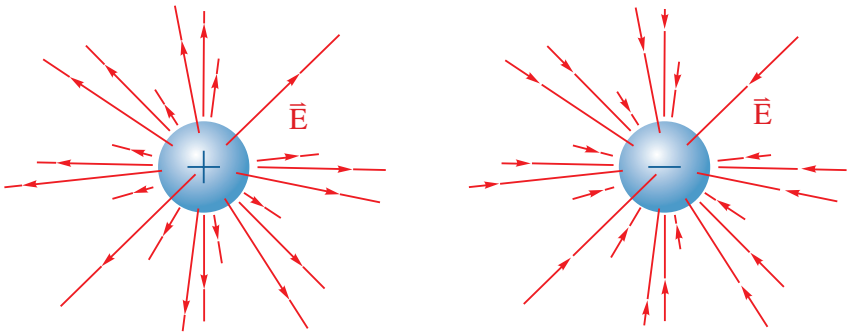
$$\vec{E} = k_0 \frac{q}{r^2} \hat{r} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

where $\hat{r}$ is a unit vector pointing from a source point to a field point.

- Electric force on a charge q placed in an electric field $\vec{E}$ is given by

$$\vec{F}_E = q\vec{E}$$

- Electric field is directed radially away from positive charges and radially toward negative charges, as shown in the following figure:



- Electric field due to a group of point charges can be determined by the superposition principle, that is—the resultant electric field at any point is the vector sum of all the fields due to every charge at source points, as follows:

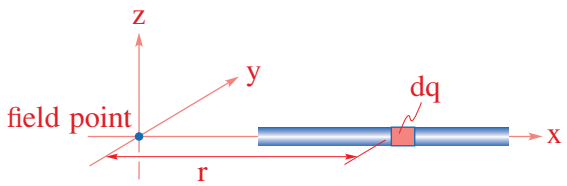
$$\vec{E} = k_0 \sum_{i=1}^{n} \frac{q_i}{r_i^2} \hat{r}_i$$

- For continuously distributed charge, the electric field at any point is given by

$$\vec{E} = k_0 \int \frac{dq}{r^2} \hat{r}$$

where dq is the charge on an infinitesimal element and

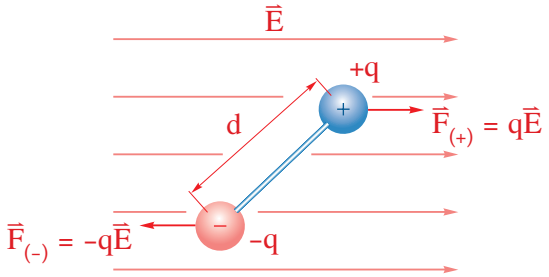
r is the distance between the element and the field point as in the figure:



Electric field lines visualize an electric field in any region of space. The number of electric field lines per unit area perpendicular to the electric field is proportional to the field's magnitude in that region.

- Electric field lines start on positive charges and end on negative charges.
- The direction of an electric field at any point is tangent to the electric field line at that point and directed along with the arrowhead of the field line.
- The magnitude of an electric field in a region with higher density of electric field lines is greater than that in other regions of less density.

An electric dipole is a pair of point charges with the same magnitude but opposite signs (+q and –q), separated by a distance d.



- An electric dipole moment $\vec{p}$ has magnitude p = qd and is directed from negative charge –q to positive charge +q:

$$\vec{p} = q\vec{r}_{(- \text{ to } +)}$$

- An electric dipole in an electric field $\vec{E}$ experiences a torque $\vec{\tau} = \vec{p} \times \vec{E}$.

"Electric Charge and Electric Force due to Point Charges"

Problem 17.1

What number of electrons is in a charge of −45.0 nC?

Solution

An electron has a charge of -1.602×10^{-19} C, so the number of the electrons in the charge −45.0 nC = -45.0×10^{-9} C is

$$\text{number of electrons} = -45.0 \times 10^{-9}\ \text{C} \times \frac{1\ \text{electron}}{-1.602 \times 10^{-19}\ \text{C}}$$

$$= 2.81 \times 10^{11}\ \text{electrons}$$ Ans

Remarks

For the precision to three significant figures, we use the charge of an electron equal to -1.602×10^{-19} C.

Problem 17.2

A group of electrons has a mass of 10.0 kg. Find the total charge of this electron group.

Solution

An electron has a mass of 9.11×10^{-31} kg. Therefore, the number of the electrons with mass 10.0 kg can be determined as follows:

$$\text{number of electrons} = 10.0\ \text{kg} \times \frac{1\ \text{electron}}{9.11 \times 10^{-31}\ \text{kg}}$$

$$= \frac{10.0}{9.11 \times 10^{-31}}\ \text{electrons}$$

Since the charge of an electron equals -1.60×10^{-19} C, the total charge of the above electrons is given by

$$\text{total charge} = \frac{10.0}{9.11 \times 10^{-31}} \text{ electrons} \times \frac{-1.60 \times 10^{-19} \text{ C}}{1 \text{ electron}}$$

$$= -1.76 \times 10^{12} \text{ C}$$ Ans

Problem 17.3

What number of electrons must be removed from a neutral object to leave it a net charge of +73 nC? Determine the decrease in mass of the object.

Solution

Since the charge of an electron is -1.602×10^{-19} C, removing one electron from a neutral object leaves it a net positive charge $+1.602 \times 10^{-19}$ C. In this case, we need the net positive charge of +73.0 nC = $+73.0 \times 10^{-19}$ C, so the number of the removed electrons is given by

$$\text{number of electrons} = +73.0 \times 10^{-9} \text{ C} \times \frac{1 \text{ electron removed}}{+1.602 \times 10^{-19} \text{ C}}$$

$$= 4.557 \times 10^{11} \text{ electrons}$$

$$= 4.56 \times 10^{11} \text{ electrons}$$ Ans

An electron has a mass of 9.109×10^{-31} kg, so 4.557×10^{11} electrons removed from the object decrease the object's mass equal to

$$\text{decrease in mass} = 4.557 \times 10^{11} \text{ electrons} \times \frac{9.109 \times 10^{-31} \text{ kg}}{1 \text{ electron}}$$

$$= 4.15 \times 10^{-19} \text{ kg}$$ Ans

Problem 17.4

Charges of 8.00 μC and −15.0 μC are on two small identical conducting spheres, placed 0.450 m apart.

1) Find the electric force exerted on each sphere.

2) Connect a conducting wire to both spheres, what is the change in the electric force on the spheres?

Solution

1) Since the charges on the two spheres have opposite signs, the electric forces on them are attractive. The magnitude of the electric force is calculated by

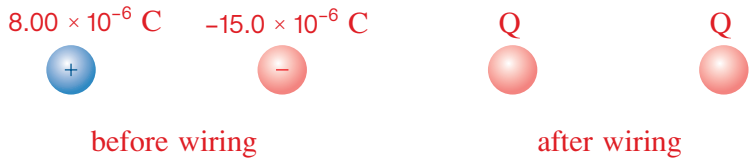
$$F_E = k_0 \frac{|q_1 q_2|}{r^2}$$

$$= 8.99 \times 10^9 \frac{\left|(8.00 \times 10^{-6})(-15.0 \times 10^{-6})\right|}{0.450^2}$$

$$F_E = 5.33 \text{ N}$$

It can be concluded that the electric force on each sphere is attractive and of magnitude 5.33 N. Ans

2) Transfer of charge occurs as connecting the conducting wire to the spheres. Since the spheres are identical, the transfer is completed on the state with the same charge on each sphere. Given the charge on each sphere at that state be a charge Q as shown in the figure. Begin by finding charge Q, then use it to calculate the electric forces on the spheres in the next step.



Charge Q can be obtained by applying the conservation of electric charges for a system consisting of the two spheres. From the figure, before wiring, the total charge of the system is

$$\text{total charge}_{\text{before}} = 8.00 \times 10^{-6} + (-15.0 \times 10^{-6})\ \text{C}$$

And after wiring, the system has the total charge of

$$\text{total charge}_{\text{after}} = Q + Q = 2Q$$

According to the charge conservation, the total charge of the system must be constant, therefore

$$\text{total charge}_{\text{before}} = \text{total charge}_{\text{after}}$$

$$8.00 \times 10^{-6} + (-15.0 \times 10^{-6}) = 2Q$$

$$Q = -3.50 \times 10^{-6}\ \text{C}$$

The result of Q indicates that the charge on each sphere after wiring is equal to -3.50×10^{-6} C, which is of the same sign, so the electric force on each sphere is altered to a repulsive force. Again, the magnitude of the repulsive force is computed by

$$F_E = k_0 \frac{|q_1 q_2|}{r^2}$$

$$= 8.99 \times 10^9 \frac{\left|(-3.50 \times 10^{-6})(-3.50 \times 10^{-6})\right|}{0.450^2}$$

$$F_E = 0.544\ \text{N}$$

In conclusion, the electric force on each sphere after wiring is repulsive and of magnitude 0.544 N. Ans

Problem 17.5

The electric force between charges $q_1 = 15.0$ nC and $q_2 = -25.0$ nC has a magnitude of 5.40×10^{-5} N. What is the distance between the charges?

Solution

The problem gives $q_1 = 15.0$ nC $= 15.0 \times 10^{-9}$ C, $q_2 = -25.0$ nC $= -25.0 \times 10^{-9}$ C and the electric force's magnitude $F_E = 5.40 \times 10^{-5}$ N. Therefore, the distance r between the charges can be obtained by the following equation:

$$F_E = k_0 \frac{|q_1 q_2|}{r^2}$$

$$r = \sqrt{\frac{k_0 |q_1 q_2|}{F_E}}$$

$$= \sqrt{\frac{8.99 \times 10^9 \left|15.0 \times 10^{-9} \times (-25.0 \times 10^{-9})\right|}{5.40 \times 10^{-5}}}$$

$$r = 0.250 \text{ m}$$ Ans

Problem 17.6

An atom has a charge +30e within its nucleus and an electron orbiting at a distance of 1.4×10^{-12} m from the nucleus. What is the magnitude of the electric force between the nucleus and electron?

Solution

The nucleus of charge $q_1 = +30e = +30 \times 1.60 \times 10^{-19}$ C and the orbiting electron of charge $q_2 = -e = -1.60 \times 10^{-19}$ C are at distance $r = 1.4 \times 10^{-12}$ m, so the magnitude of the electric force between these two charges is given by

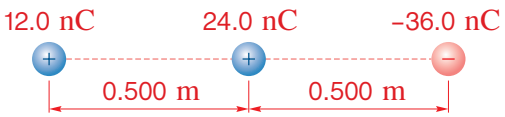
$$F_E = k_0 \frac{|q_1 q_2|}{r^2}$$

$$= 8.99 \times 10^9 \times \frac{|(+30 \times 1.60 \times 10^{-19})(-1.60 \times 10^{-19})|}{(1.4 \times 10^{-12})^2}$$

$$F_E = 3.5 \times 10^{-3}\ \text{N}$$ Ans

Problem 17.7

Three point charges are arranged in a straight line, equally spaced 0.500 m, as shown in Fig. (a). Find the net electric force exerted on the middle charge.



(a)

Solution

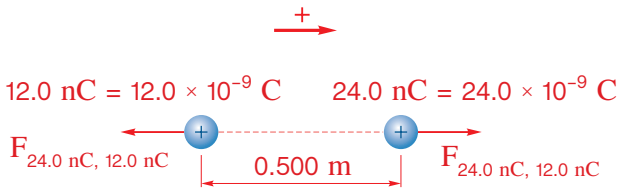
The net electric force on the middle charge 24.0 nC is the vector sum of the electric forces on this charge due to charges 12.0 nC and –36.0 nC:

$$\vec{F}_{24.0\ \text{nC}} = \vec{F}_{24.0\ \text{nC},12.0\ \text{nC}} + \vec{F}_{24.0\ \text{nC},-36.0\ \text{nC}} \quad \ldots (1)$$

Consider charges 24.0 nC and 12.0 nC in Fig. (b). Since these two charges are of the same sign, the electric forces are repulsive. On charge 24.0 nC, the electric force $\vec{F}_{24.0\ \text{nC},12.0\ \text{nC}}$ is directed to the right. Set the right direction as a positive direction. Therefore, the electric force $\vec{F}_{24.0\ \text{nC},12.0\ \text{nC}}$ can be determined as follows

$$\vec{F}_{24.0\ \text{nC},12.0\ \text{nC}} = +k_0 \frac{|q_1 q_2|}{r^2}$$

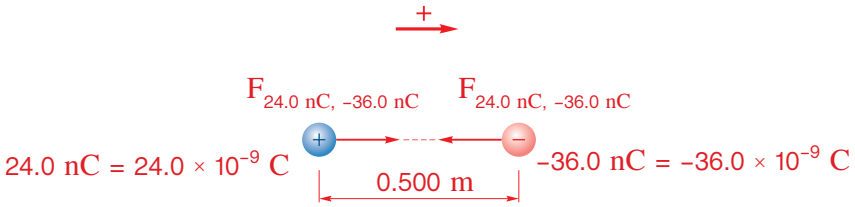
$$\vec{F}_{24.0\ \text{nC},12.0\ \text{nC}} = +k_0 \frac{|(24.0 \times 10^{-9})(12.0 \times 10^{-9})|}{0.500^2} \quad \ldots (2)$$



(b)

Likewise, consider the electric force between charges 24.0 nC and −36.0 nC in Fig. (c). It can be seen that the forces are attractive because the charges have opposite signs. On charge 24.0 nC, the electric force $\vec{F}_{24.0\text{ nC},-36.0\text{ nC}}$ is directed to the right (positive direction) and given by

$$\vec{F}_{24.0\text{ nC},-36.0\text{ nC}} = +k_0 \frac{\left|(24.0 \times 10^{-9})(-36.0 \times 10^{-9})\right|}{0.500^2} \qquad \ldots (3)$$



(c)

Substitute the forces $\vec{F}_{24.0\text{ nC},12.0\text{ nC}}$ and $\vec{F}_{24.0\text{ nC},-36.0\text{ nC}}$ from Eqs. (2) and (3) into Eq. (1) to get the net electric force on charge 24.0 nC:

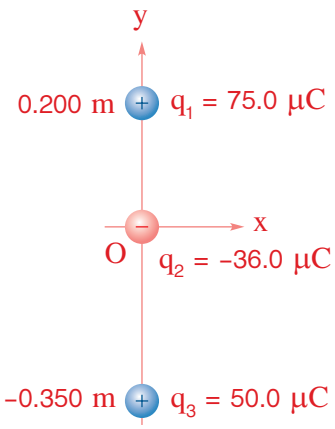
$$\vec{F}_{24.0\text{ nC}} = \left(+k_0 \frac{\left|24.0 \times 10^{-9} \times 12.0 \times 10^{-9}\right|}{0.500^2}\right) + \left(+k_0 \frac{\left|24.0 \times 10^{-9} \times (-36.0 \times 10^{-9})\right|}{0.500^2}\right)$$

$$= +4.14 \times 10^{-5}\text{ N}$$

$$\vec{F}_{24.0\text{ nC}} = 4.14 \times 10^{-5}\text{ N directed to the right} \qquad \text{Ans}$$

Problem 17.8

Three point charges are located on the y-axis, as shown in Fig. (a). Find the resultant electric force on charge q_2.



(a)

Solution

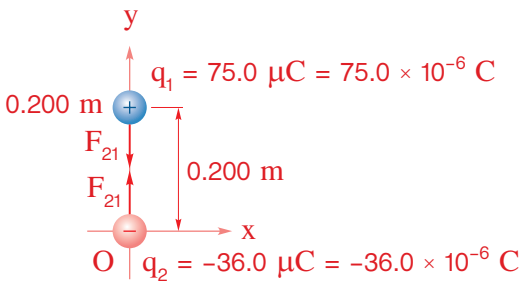
The resultant electric force on charge q_2 is the vector sum of the forces exerted on charge q_2 due to charges q_1 and q_3:

$$\vec{F}_2 = \vec{F}_{21} + \vec{F}_{23} \qquad \ldots (1)$$

Figure (b) shows the electric forces between charges q_1 and q_2. Since the charges have opposite signs, the electric forces between them are attractive. On charge q_2, the electric force $\vec{F}_{21}$ is in the positive y-direction $(+\vec{j})$. Charge q_1 is on the y-axis at y = 0.200 m and charge q_2 is at the origin (y = 0). So the distance between q_1 and q_2 is 0.200 m, and the force $\vec{F}_{21}$ is given by

$$\vec{F}_{21} = k_0 \frac{|q_1 q_2|}{r^2}(+\vec{j})$$

$$\vec{F}_{21} = k_0 \frac{\left|75.0 \times 10^{-6} \times (-36.0 \times 10^{-6})\right|}{0.200^2}(+\vec{j}) \qquad \ldots (2)$$

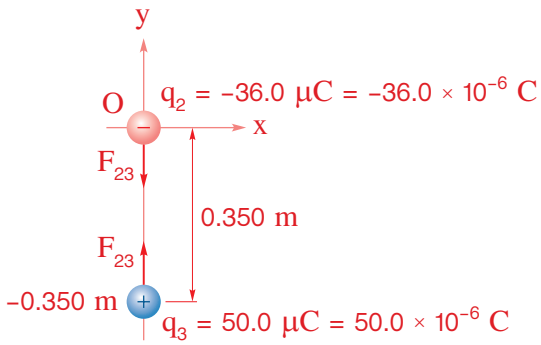


(b)

In the same way, charges q_2 and q_3 have opposite signs, so the electric forces between them are attractive, as in Fig. (c). From the figure, force $\vec{F}_{23}$ on charge q_2 is in the negative y-direction $(-\vec{j})$, and the distance between the charges is 0.350 m. Therefore, the electric force $\vec{F}_{23}$ can be calculated as follows:

$$\vec{F}_{23} = k_0 \frac{|q_2 q_3|}{r_{23}^2}(-\vec{j})$$

$$\vec{F}_{23} = k_0 \frac{\left|(-36.0 \times 10^{-6}) \times 50.0 \times 10^{-6}\right|}{0.350^2}(-\vec{j}) \qquad \text{... (3)}$$



(c)

Substituting $\vec{F}_{21}$ and $\vec{F}_{23}$ from Eqs. (2) and (3) into Eq. (1), and using $k_0 = 8.99 \times 10^9\ N{\cdot}m^2/C^2$ gives the resultant electric force on charge q_2:

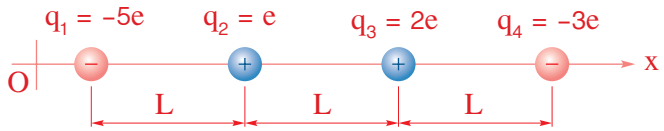
$$\vec{F}_2 = k_0 \frac{\left|75.0 \times 10^{-6} \times (-36.0 \times 10^{-6})\right|}{0.200^2}(+\vec{j}) + k_0 \frac{\left|(-36.0 \times 10^{-6}) \times 50.0 \times 10^{-6}\right|}{0.350^2}(-\vec{j})$$

$$= +475\vec{j}\ \text{N}$$

$\vec{F}_2$ = 475 N in the positive y-direction Ans

Problem 17.9

In Fig. (a), find the resultant electric force on charge q_3 in terms of e, distance L, and unit vector $\vec{i}$.



(a)

Solution

The resultant electric force on charge q_3 is the vector sum of the forces on charge q_3 exerted by charges q_1, q_2, and q_4, as follows:

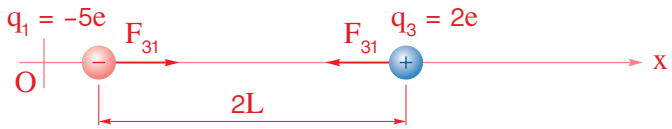
$$\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32} + \vec{F}_{34} \quad \text{... (1)}$$

Figure (b) shows electric forces between charges q_3 and q_1. Since the charges are of opposite signs, the electric forces are attractive, and force $\vec{F}_{31}$ on charge q_3 is in the negative x-direction $(-\vec{i})$. Charges q_1 and q_3 are at distance 2L apart, so the force $\vec{F}_{31}$ is obtained by

$$\vec{F}_{31} = k_0 \frac{|q_1 q_3|}{r_{13}^2}(-\vec{i})$$

$$= k_0 \frac{|(-5e) \times 2e|}{(2L)^2}(-\vec{i})$$

$$\vec{F}_{31} = -\frac{5}{2}\frac{k_0 e^2}{L^2}\vec{i} \quad \text{... (2)}$$



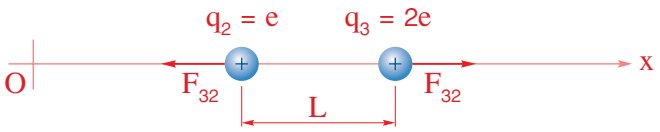
(b)

In Fig. (c), charges q_2 and q_3 have the same sign, therefore, the electric force on charge q_3 is repulsive and directed along the positive x-axis $(+\vec{i})$. Electric force $\vec{F}_{32}$ on charge q_3 due to charge q_2, a distance L apart, can be determined as follows:

$$\vec{F}_{32} = k_0 \frac{|q_2 q_3|}{r_{23}^2}(+\vec{i})$$

$$= k_0 \frac{|e \times 2e|}{L^2}(+\vec{i})$$

$$\vec{F}_{32} = 2\frac{k_0 e^2}{L^2}\vec{i} \qquad \ldots (3)$$



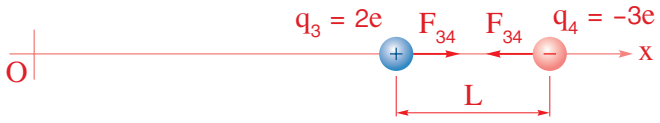
(c)

Considering charges q_3 and q_4 in Fig. (d), this pair of charges have opposite signs, so electric force $\vec{F}_{34}$ on charge q_3 is attractive in the positive x-direction $(+\vec{i})$ and found by

$$\vec{F}_{34} = k_0 \frac{|q_3 q_4|}{r_{34}^2}(+\vec{i})$$

$$= k_0 \frac{|2e \times (-3e)|}{L^2}(+\vec{i})$$

$$\vec{F}_{34} = 6\frac{k_0 e^2}{L^2}\vec{i} \qquad \ldots (4)$$



(d)

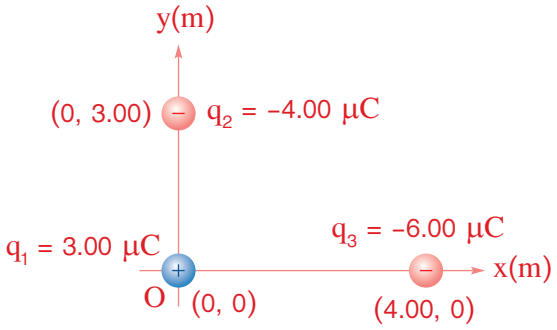
Substituting forces $\vec{F}_{31}$, $\vec{F}_{32}$, and $\vec{F}_{34}$ from Eqs. (2) through (4) into Eq. (1) leads to

$$\vec{F}_3 = -\frac{5}{2}\frac{k_0 e^2}{L^2}\vec{i} + 2\frac{k_0 e^2}{L^2}\vec{i} + 6\frac{k_0 e^2}{L^2}\vec{i}$$

$$\vec{F}_3 = \frac{11}{2}\frac{k_0 e^2}{L^2}\vec{i}$$ Ans

Problem 17.10

Three point charges are placed at the corners of a triangle indicated by coordinates (x, y), as shown in Fig. (a). Find the resultant electric force on charge q_2 due to charges q_1 and q_3 in terms of unit vectors $\vec{i}$ and $\vec{j}$, and also determine the magnitude and direction of the resultant force.



(a)

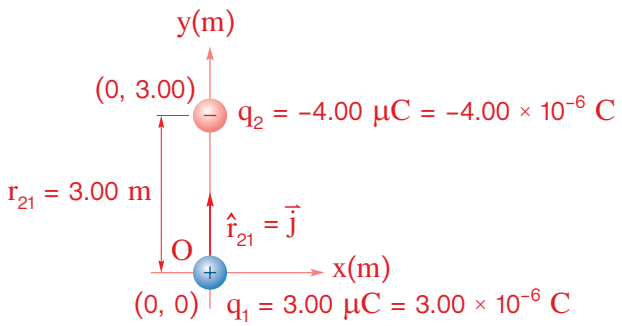
Solution

The resultant electric force on charge q_2 is the vector sum of all the forces exerted on charge q_2 due to charges q_1 and q_3:

$$\vec{F}_2 = \vec{F}_{21} + \vec{F}_{23} \quad \ldots (1)$$

Here, we would find electric forces $\vec{F}_{21}$ and $\vec{F}_{23}$ in vector form. Begin with force $\vec{F}_{21}$ as follows:

$$\vec{F}_{21} = k_0 \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21} \quad \text{... (2)}$$



(b)

In Fig. (b), we get

- $q_1 = 3.00\ \mu C = 3.00 \times 10^{-6}$ C
- $q_2 = -4.00\ \mu C = -4.00 \times 10^{-6}$ C
- r_{21} is the distance between charges q_1 and q_2. Since charge q_1 is at coordinates (0, 0) and charge q_2 (0, 3.00 m), the charges are 3.00 m apart, that is $r_{21} = 3.00$ m.
- $\hat{r}_{21}$ is a unit vector to position charge q_2 relative to charge q_1. As observing from charge q_1, this vector is the unit vector directed along the positive y-axis: $\hat{r}_{21} = \vec{j}$.

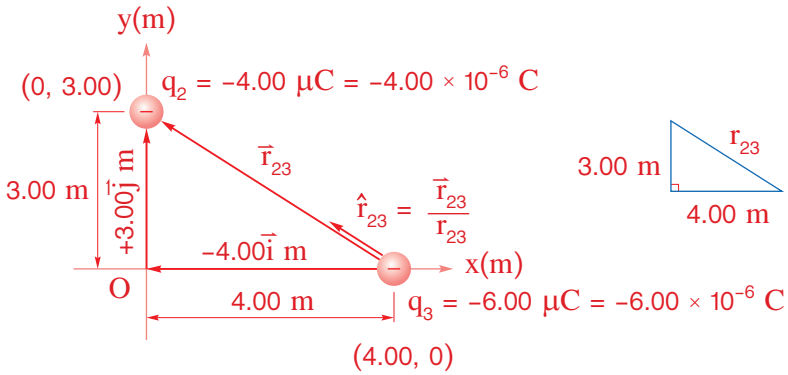
Substitute all the above quantities into Eq. (2):

$$\vec{F}_{21} = k_0 \frac{(3.00 \times 10^{-6})(-4.00 \times 10^{-6})}{3.00^2} (\vec{j})$$

$$\vec{F}_{21} = -\frac{4}{3} k_0 \times 10^{-12} \vec{j} \quad \text{... (3)}$$

Electric force $\vec{F}_{23}$ is written in vector form as

$$\vec{F}_{23} = k_0 \frac{q_2 q_3}{r_{23}^2} \hat{r}_{23} \quad \text{... (4)}$$



(c)

Figure (c) tells us that

- $q_2 = -4.00\ \mu C = -4.00 \times 10^{-6}$ C
- $q_3 = -6.00\ \mu C = -6.00 \times 10^{-6}$ C
- r_{23} is the distance between charges q_2 and q_3, which is the hypotenuse of the right triangle in Fig. (c). By applying the Pythagorean Theorem, the relationship for this triangle is written as

$$r_{23}^2 = 3.00^2 + 4.00^2$$

$$r_{23} = \sqrt{3.00^2 + 4.00^2} = 5.00 \text{ m}$$

- Similar to unit vector $\hat{r}_{21}$, unit vector $\hat{r}_{23}$ describes the position of charge q_2 by an observer at charge q_3 and equals

$$\hat{r}_{23} = \frac{\vec{r}_{23}}{r_{23}}$$

In Fig. (c), vector $\vec{r}_{23}$, pointing from charge q_3 to q_2, is written in x- and y-components as follows:

$$\vec{r}_{23} = -4.00\vec{i} + 3.00\vec{j} \text{ m}$$

Substitute q_2, q_3, r_{23}, and $\hat{r}_{23}$ into Eq. (4):

$$\vec{F}_{23} = k_0 \frac{q_2 q_3}{r_{23}^2}\left(\frac{\vec{r}_{23}}{r_{23}}\right)$$

$$= k_0 \frac{q_2 q_3}{r_{23}^3}\,\vec{r}_{23}$$

$$= k_0 \frac{(-4.00 \times 10^{-6})(-6.00 \times 10^{-6})}{5.00^3}(-4.00\,\vec{i} + 3.00\,\vec{j})$$

$$\vec{F}_{23} = -\frac{96}{125} k_0 \times 10^{-12}\,\vec{i} + \frac{72}{125} k_0 \times 10^{-12}\,\vec{j} \qquad \text{... (5)}$$

Substituting the values of $\vec{F}_{21}$ and $\vec{F}_{23}$ from Eqs. (3) and (5) into Eq. (1) and using $k_0 = 8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$ gives

$$\vec{F}_2 = \left(-\frac{4}{3} k_0 \times 10^{-12}\,\vec{j}\right) + \left(-\frac{96}{125} k_0 \times 10^{-12}\,\vec{i} + \frac{72}{125} k_0 \times 10^{-12}\,\vec{j}\right)$$

$$\vec{F}_2 = -6.90 \times 10^{-3}\,\vec{i} - 6.81 \times 10^{-3}\,\vec{j}\ \text{N} \qquad \text{Ans}$$

The resultant electric force $\vec{F}_2$ reveals that this force has a horizontal component of 6.90×10^{-3} N along the negative x-axis and a vertical component of 6.81×10^{-3} N along the negative y-axis, as shown in Fig. (d). Therefore, the magnitude and direction of the force $\vec{F}_2$ can be determined as follows:

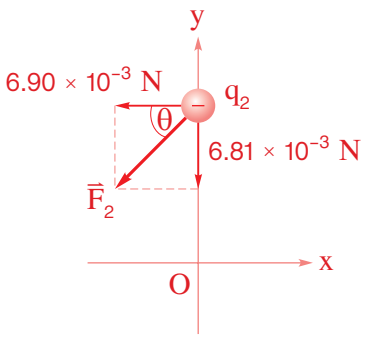
$$F_2 = \sqrt{(6.90 \times 10^{-3})^2 + (6.81 \times 10^{-3})^2}$$

$$F_2 = 9.69 \times 10^{-3}\ \text{N}$$

$$\tan\theta = \frac{6.81 \times 10^{-3}}{6.90 \times 10^{-3}}$$

$$\theta = \tan^{-1}\left(\frac{6.81 \times 10^{-3}}{6.90 \times 10^{-3}}\right) = 44.6^\circ$$

It can be concluded that the resultant electric force on charge q_2 has a magnitude of 9.69×10^{-3} N and makes a 44.6° angle below the negative x-axis. Ans



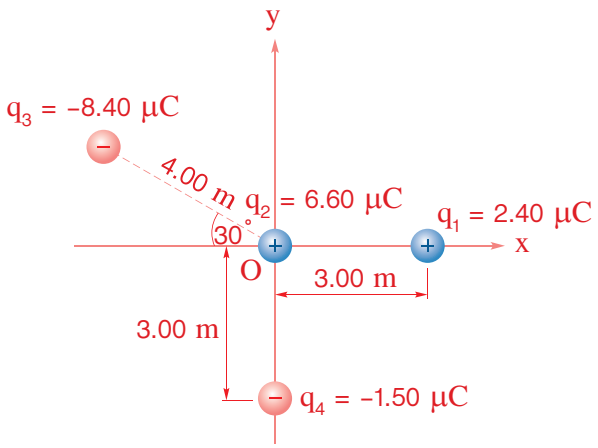
(d)

Remarks

It would be best for your mathematics skill to determine the magnitude and direction of electric forces $\vec{F}_{21}$ and $\vec{F}_{23}$ separately.

Problem 17.11

What are the magnitude and direction of the resultant electric force on charge q_2 due to charges q_1, q_3, and q_4 in Fig. (a)?



(a)

Solution

The resultant electric force on charge q_2 is the vector sum of the forces exerted on q_2 due to charges q_1, q_3 and q_4:

$$\vec{F}_2 = \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{24} \quad \text{... (1)}$$

We would find the magnitude and direction of electric forces $\vec{F}_{21}$, $\vec{F}_{23}$ and $\vec{F}_{24}$ separately, as follows:

Figure (b) represents electric force $\vec{F}_{21}$ on charge q_2 due to charge q_1. Since these two charges are of the same sign, the force $\vec{F}_{21}$ is repulsive along the negative x–axis. The magnitude of the force $\vec{F}_{21}$ is given by

$$F_{21} = k_0 \frac{|q_1 q_2|}{r_{21}^2}$$

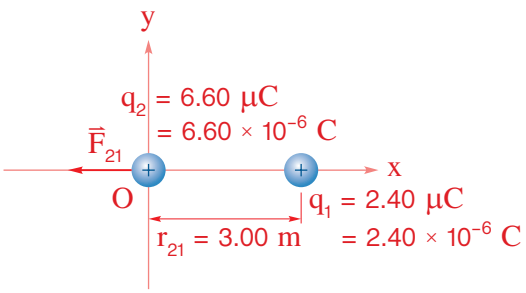
$$= k_0 \frac{|2.40 \times 10^{-6} \times 6.60 \times 10^{-6}|}{3.00^2}$$

$$F_{21} = 1.76 \times 10^{12} k_0$$

Therefore,

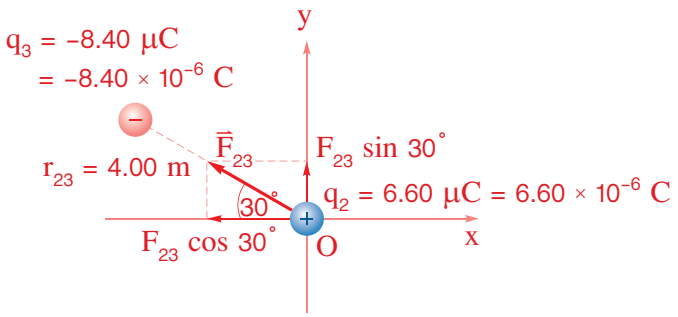
$$\vec{F}_{21} = 1.76 \times 10^{-12} k_0 (-\vec{i})$$

$$\vec{F}_{21} = -1.76 \times 10^{-12} k_0 \vec{i} \quad \text{... (2)}$$



(b)

Since charges q_2 and q_3 are of opposite signs, the electric force between the charges is attractive. In Fig. (c), electric force $\vec{F}_{23}$ on charge q_2 due to charge q_3 is shown. Force $\vec{F}_{23}$ is directed from charges q_2 to q_3 along the line connecting the charges.



(c)

The magnitude of the force $\vec{F}_{23}$ is

$$F_{23} = k_0 \frac{|q_2 q_3|}{r_{23}^2}$$

$$= k_0 \frac{|6.60 \times 10^{-6} \times (-8.40 \times 10^{-6})|}{4.00^2}$$

$$F_{23} = 3.465 \times 10^{-12}\, k_0$$

Force $\vec{F}_{23}$ makes a 30° angle above the negative x-axis, so the x- and y-components of the force are shown in Fig. (c), and can be written in the component form, as follows:

$$\vec{F}_{23} = F_{23} \cos 30^\circ (-\vec{i}) + F_{23} \sin 30^\circ (+\vec{j})$$

$$\vec{F}_{23} = -F_{23} \cos 30^\circ \vec{i} + F_{23} \sin 30^\circ \vec{j}$$

Substitute $F_{23} = 3.465 \times 10^{-12}\, k_0$ into the above expression:

$$\vec{F}_{23} = -3.465 \times 10^{-12} k_0 \cos 30^\circ \vec{i} + 3.465 \times 10^{-12} k_0 \sin 30^\circ \vec{j} \qquad \ldots (3)$$

In Fig. (d), force $\vec{F}_{24}$ is the electric force on charge q_2 due to charge q_4. Because the charges have opposite signs, the electric force is attractive, and thr force $\vec{F}_{24}$ is directed downward along the negative y-axis. The magnitude of the force is given by

$$F_{24} = k_0 \frac{|q_2 q_4|}{r_{24}^2}$$

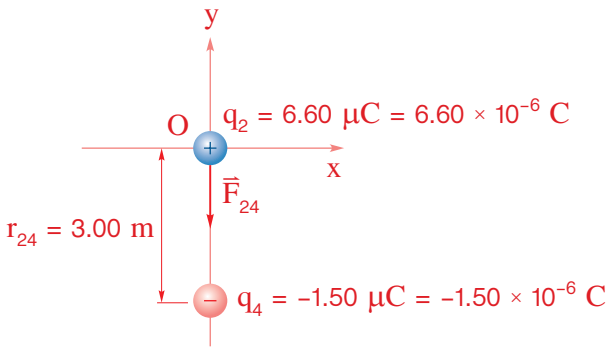
$$= k_0 \frac{|6.60 \times 10^{-6} \times (-1.50 \times 10^{-6})|}{3.00^2}$$

$$F_{24} = 1.10 \times 10^{-12} k_0$$

Therefore,

$$\vec{F}_{24} = 1.10 \times 10^{-12}\, k_0 (-\vec{j})$$

$$\vec{F}_{24} = -1.10 \times 10^{-12} k_0\, \vec{j} \qquad \text{... (4)}$$



(d)

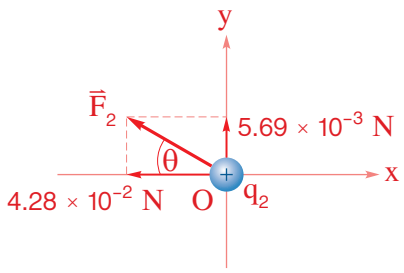
Substituting forces $\vec{F}_{21}, \vec{F}_{23}$, and $\vec{F}_{24}$ from Eqs. (2) through (4) into Eq. (1) and using $k_0 = 8.99 \times 10^9$ N·m^2/C^2 leads to

$$\vec{F}_2 = (-1.76 \times 10^{-12} k_0\, \vec{i}\,) + (-3.465 \times 10^{-12} k_0 \cos 30^\circ\, \vec{i}\, +$$

$$3.465 \times 10^{-12} k_0 \sin 30^\circ\, \vec{j}\,) + (-1.10 \times 10^{-12} k_0\, \vec{j}\,)$$

$$\vec{F}_2 = -4.28 \times 10^{-2}\, \vec{i} + 5.69 \times 10^{-3}\, \vec{j} \quad \text{N}$$

Sketch the force $\vec{F}_2$ in the x–y coordinate system, as in Fig. (e). It is found that the force has a horizontal component of 4.28×10^{-2} N along the negative x-axis and a vertical component of 5.69×10^{-3} N along the positive y-axis. Therefore, the magnitude and direction of the force $\vec{F}_2$ are calculated by



(e)

$$F_2 = \sqrt{(4.28 \times 10^{-2})^2 + (5.69 \times 10^{-3})^2}$$

$$F_2 = 4.32 \times 10^{-2} \text{ N}$$

$$\tan \theta = \frac{5.69 \times 10^{-3}}{4.28 \times 10^{-2}}$$

$$\theta = \tan^{-1}\left(\frac{5.69 \times 10^{-3}}{4.28 \times 10^{-2}}\right) = 7.57^\circ$$

In conclusion, the resultant electric force on charge q_2 has a magnitude of 4.32×10^{-2} N and makes a 7.57° angle above the negative x-axis. Ans

Problem 17.12

Two point charges are placed 16.0 m apart, then displace them until the electric force between the charges increases four times the original value. Find the distance between the charges after changing their positions.

Solution

The following relationship gives the magnitude of an electric force between charges:

$$F_E = k_0 \frac{|q_1 q_2|}{r^2} \quad \text{... (1)}$$

Since charges q_1 and q_2 are constant during changing their positions, we can rewrite Eq. (1) for this problem, as follows:

$$F_E = k_0 \frac{|q_1 q_2|}{r^2}$$

$$F_E = \frac{C}{r^2} \qquad \text{... (2)}$$

where $C = k_0|q_1 q_2| = \text{constant}$

Use Eq. (2) for the magnitude of the electric force before and after changing the positions of the charges, as follows:

before displacing : $r_1 = 16.0$ m, $F_{E1} = F$

$$F = \frac{C}{(16.0)^2} \qquad \text{... (3)}$$

after displacing : $r_2 = r$, $F_{E2} = 4F$

$$4F = \frac{C}{r^2} \qquad \text{... (4)}$$

Divide Eq. (3) by Eq. (4):

$$\frac{F}{4F} = \frac{\frac{C}{(16.0)^2}}{\frac{C}{r^2}} = \left(\frac{r}{16.0}\right)^2$$

$$\frac{1}{2} = \frac{r}{16.0}$$

$$r = \frac{16.0}{2} = 8.00 \text{ m}$$

In conclusion, after displacing, the charges are 8.00 m apart, and the magnitude of the electric force increases four times. Ans

Problem 17.13

An electric force between two point charges has a magnitude of 4.5×10^{-3} N. If displacing them until the distance between the charges is $\frac{1}{4}$ times the original value, find the magnitude of the electric force.

Solution

In the previous problem, the expression for magnitude F_E of an electric force, a distance r apart, and constant charges q_1 and q_2, is

$$F_E = \frac{C}{r^2} \qquad ...(1)$$

where $C = k_0|q_1q_2| = \text{constant}$

Apply Eq. (1) for the magnitude of the electric force in this problem both before and after changing their positions, as follows:

before displacing: $F_{E1} = 4.5 \times 10^{-3}$ N, $r_1 = r$

$$4.5 \times 10^{-3} = \frac{C}{r^2} \qquad ... (2)$$

after displacing: $F_{E2} = F_E$, $r_2 = \frac{1}{4}r$

$$F_E = \frac{C}{\left(\frac{1}{4}r\right)^2} \qquad ... (3)$$

Divide Eq. (3) by Eq. (2):

$$\frac{F_E}{4.5 \times 10^{-3}} = \frac{\frac{C}{\left(\frac{1}{4}r\right)^2}}{\frac{C}{r^2}} = 4^2$$

$$F_E = 4^2 \times 4.5 \times 10^{-3} = 7.2 \times 10^{-2} \text{ N}$$

It can be concluded that the magnitude of the electric force increases to 7.2×10^{-2} N. Ans

Problem 17.14

Each of two small spheres has positive charges, in which the charge on one sphere is five times that on the other. If the electric force between the spheres, a distance 30.0 cm apart, is of magnitude 1.25 N, find the electric charge on each sphere.

Solution

We are given "a sphere has a charge five times that on the other," so set $q_1 = Q$ and $q_2 = 5q_1 = 5Q$. Moreover, the electric force between them, a distance $r = 30.0$ cm $= 30.0 \times 10^{-2}$ cm, has a magnitude of $F_E = 1.25$ N. For such the data, apply the expression for the magnitude of an electric force, as follows:

$$F_E = k_0 \frac{|q_1 q_2|}{r^2} \qquad \text{... (1)}$$

Since charges q_1 and q_2 are positive, the absolute value sign in Eq. (1) can be removed:

$$F_E = k_0 \frac{q_1 q_2}{r^2} \qquad \text{... (2)}$$

Substitute all the quantities into Eq. (2) to find charge Q:

$$1.25 = \frac{8.99 \times 10^9 \times Q \times 5Q}{(30.0 \times 10^{-2})^2}$$

$$Q = 1.58 \times 10^{-6} \text{ C}$$

In conclusion, the two spheres have charges of $Q = 1.58 \times 10^{-6}$ C and $5Q = 5 \times 1.582 \times 10^{-6} = 7.91 \times 10^{-6}$ C. Ans

Problem 17.15

Two small neutral spheres are 25 cm apart. If removing charge Q from a sphere then put it into the other, the magnitude of the electric force between the spheres is 8.0×10^{-3} N. What is the number of the electrons in charge Q?

Solution

Begin by finding charge Q producing the electric force of magnitude 8.0×10^{-3} N, then use it to calculate the number of the electrons by applying the fact that the charge of an electron has a magnitude of 1.60×10^{-19} C.

When transferring charge Q from one sphere into the second, the charges on the two spheres are –Q and +Q. Since the charges, –Q and +Q, have opposite signs, the electric force between them is attractive. The magnitude of the force is given by

$$F_E = k_0 \frac{|q_1 q_2|}{r^2}$$

$$= k_0 \frac{|(-Q) \times Q|}{r^2}$$

$$F_E = k_0 \frac{Q^2}{r^2} \qquad \ldots (1)$$

The problem gives $F_E = 8.0 \times 10^{-3}$ N and $r = 25$ cm $= 25 \times 10^{-2}$ m; substitute them into Eq. (1) to find charge Q:

$$8.0 \times 10^{-3} = 8.99 \times 10^9 \times \frac{Q^2}{(25 \times 10^{-2})^2}$$

$$Q = 2.358 \times 10^{-7} \text{ C}$$

Since an electron has a charge of magnitude 1.60×10^{-19} C, the number of the electrons in $Q = 2.358 \times 10^{-7}$ C is obtained by

$$\text{number of electrons} = 2.358 \times 10^{-7} \text{ C} \times \frac{1 \text{ electron}}{1.60 \times 10^{-19} \text{ C}}$$

number of electrons = 1.47×10^{12} electrons Ans

Problem 17.16

Positive charge Q is divided into two parts, placed a distance apart. What are the charges on both parts if the magnitude of the electric force between them is maximum?

Solution

Set charge Q divided into two parts: the first part has a charge of $q_1 = q$ and the second has a charge of $q_2 = Q - q$. Given these two parts placed a distance r apart. The magnitude of the electric force between the charges is given by

$$F_E = k_0 \frac{|q_1 q_2|}{r^2}$$

$$= k_0 \frac{|q \times (Q - q)|}{r^2}$$

$$F_E = \frac{k_0}{r^2}(qQ - q^2) \quad \text{... (1)}$$

The magnitude F_E of the electric force is maximum as the first derivative of F_E with respect to charge q equals zero, so

$$\frac{dF_E}{dq} = \frac{d}{dq}\left(\frac{k_0}{r^2}(qQ - q^2)\right)$$

$$= \frac{k_0}{r^2}(Q - 2q)$$

and $$\frac{k_0}{r^2}(Q - 2q) = 0$$

$$q = \frac{Q}{2}$$

Since the result $q = \frac{Q}{2}$ may maximize or minimize the magnitude F_E, we must check it by calculating the second derivative of F_E, as follows:

$$\frac{d^2F_E}{dq^2} = \frac{d}{dq}\left(\frac{k_0}{r^2}(Q - 2q)\right)$$

$$= \frac{k_0}{r^2}(0 - 2)$$

$$\frac{d^2F_E}{dq^2} = -\frac{2k_0}{r^2}$$

Since k_0 and r^2 are positive numbers, $\frac{d^2F_E}{dq^2} < 0$ and $q = \frac{Q}{2}$ maximizes the magnitude F_E.

In conclusion, charge Q must be divided into two equal parts, each $\frac{Q}{2}$. Ans

Problem 17.17

Charge 9.0 nC is divided into two parts, placed a distance 5.0 mm apart. Find the magnitude of the electric force as maximum as possible.

Solution

In the previous problem, the magnitude of the electric force is largest when we divide charge 9.0 nC into two equal parts, that is $q_1 = q_2 = \frac{9.0}{2} = 4.5$ nC $= 4.5 \times 10^{-9}$ C. Use the data of q_1, q_2, and distance r = 5.0 mm $= 5.0 \times 10^{-3}$ m to calculate the maximum magnitude of the electric force:

$$F_{E,\,max} = k_0 \frac{|q_1 q_2|}{r^2}$$

$$= k_0 \frac{q_1 q_2}{r^2}$$

$$= 8.99 \times 10^9 \times \frac{(4.5 \times 10^{-9})^2}{(5.0 \times 10^{-3})^2}$$

$$F_{E,\,max} = 7.3 \times 10^{-3}\ \text{N}$$ Ans

Problem 17.18

Two small spheres have a total charge of 9.00 nC. The electric force between the spheres, placed a distance 4.00 mm apart, is repulsive and of magnitude 5.00×10^{-3} N. What is the charge on each sphere? If the electric force is attractive and of magnitude 5.00×10^{-3} N, find the charge on each sphere.

Solution

Repulsive electric force

Since the total charge of 9.00 nC is positive, the electric force is repulsive only when the charges on both spheres are positive. Set a charge on a sphere is q, and the total charge is $Q = 9.00\ \text{nC} = 9.00 \times 10^{-9}$ C, so

$$\begin{aligned} q_1 &= q \\ q_2 &= Q - q \\ r &= 4.00\ \text{mm} = 4.00 \times 10^{-3}\ \text{m} \\ F_E &= 5.00 \times 10^{-3}\ \text{N} \end{aligned}$$

Use the above data for the expression of an electric force's magnitude, and then find the value of q, as follows:

$$\begin{aligned} F_E &= k_0 \frac{|q_1 q_2|}{r^2} \\ &= k_0 \frac{q_1 q_2}{r^2}, \ (q_1 \text{ and } q_2 \text{ are positive}) \\ &= k_0 \frac{q \times (Q - q)}{r^2} \\ \frac{F_E r^2}{k_0} &= -q^2 + Qq \end{aligned}$$

$$q^2 - Qq + \frac{F_E r^2}{k_0} = 0$$

$$q = \frac{Q \pm \sqrt{Q^2 - 4(1)\frac{F_E r^2}{k_0}}}{2(1)}$$

$$q = 7.87 \times 10^{-9}\ \text{C},\ 1.13 \times 10^{-9}\ \text{C}$$

Substituting the values of q into $q_2 = Q - q$ gets $q_2 = 1.13 \times 10^{-9}$ C and 7.87×10^{-9} C. So, the charges on the spheres are 7.87×10^{-9} C and 1.13×10^{-9} C or 7.87 nC and 1.13 nC. Ans

Attractive electric force

In the case of the attractive electric force, the charges on the two spheres must have opposite signs. Set the charge on the first sphere is negative and of magnitude q, therefore, $q_1 = -q$. The charge q_2 of the other sphere must be positive and given by

$$q_1 + q_2 = Q$$

$$(-q) + q_2 = Q$$

$$q_2 = Q + q$$

Use the data of q_1 and q_2 for the expression of an electric force's magnitude, and then find the value of q:

$$F_E = k_0 \frac{|q_1 q_2|}{r^2}$$

$$= k_0 \frac{|(-q) \times (Q + q)|}{r^2}$$

$$= k_0 \frac{q \times (Q + q)}{r^2}$$

$$\frac{F_E r^2}{k_0} = q^2 + Qq$$

$$q^2 + Qq - \frac{F_E r^2}{k_0} = 0$$

$$q = \frac{-Q \pm \sqrt{Q^2 - 4(1)\left(-\frac{F_E r^2}{k_0}\right)}}{2(1)}$$

$$q = 8.99 \times 10^{-10}\ \text{C},\ -9.90 \times 10^{-9}\ \text{C}$$

Since q must be a positive number, $q = 8.99 \times 10^{-10}$ C $= 0.899 \times 10^{-9}$ C, and the charge on each sphere is

$$q_1 = -q = -0.899 \times 10^{-9}\ \text{C} = -0.899\ \text{nC}$$

$$q_2 = Q + q = 9.00 \times 10^{-9} + 0.899 \times 10^{-9}$$

$$= 9.90 \times 10^{-9}\ \text{C}$$

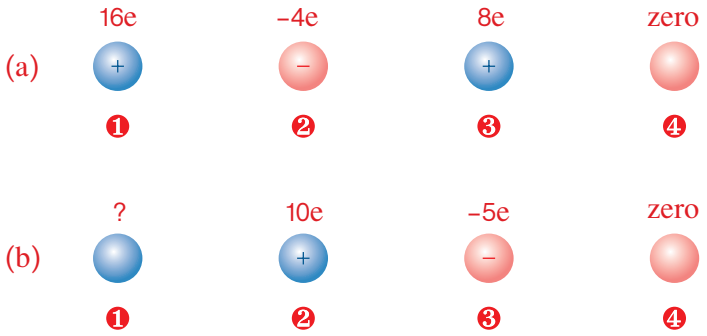
$$= 9.90\ \text{nC}$$

In conclusion, the charges on the two spheres are −0.899 nC and 9.90 nC. Ans

Problem 17.19

1) Four identical conducting spheres have charges, as shown in Fig. (a). Sphere ❹ is touched to sphere ❶, and then they are separated. Next, sphere ❹ is touched to sphere ❷, and then they are separated. Finally, sphere ❹ is touched to sphere ❸, and then they are separated. What is the final charge on sphere ❹?

2) Just as the steps in 1), if the initial charge on sphere ❶ is unknown, as in Fig. (b), and the final charge on sphere ❹ is −2e, find the initial charge on sphere ❶.



Solution

1) Apply the charge conservation on the system of two spheres that are touched each other. Since all the spheres are identical, after touching, the charge on each sphere is equal. Consider the contact step by step as follows:

Sphere ❶ is touched to sphere ❹

$$Q_{1\ \text{before}} + Q_{4\ \text{before}} = Q_{1\ \text{after}} + Q_{4\ \text{after}}$$

$$16e + 0 = Q_{4\ \text{after}} + Q_{4\ \text{after}}$$

$$Q_{4\ \text{after}} = \frac{16e}{2} = 8e$$

Sphere ❷ is touched to sphere ❹

$$Q_{2\ \text{before}} + Q_{4\ \text{before}} = Q_{2\ \text{after}} + Q_{4\ \text{after}}$$

$$(-4e) + 8e = Q_{4\ \text{after}} + Q_{4\ \text{after}}$$

$$Q_{4\ \text{after}} = \frac{4e}{2} = 2e$$

Sphere ❸ is touched to sphere ❹

$$Q_{3\ \text{before}} + Q_{4\ \text{before}} = Q_{3\ \text{after}} + Q_{4\ \text{after}}$$

$$8e + 2e = Q_{4\ \text{after}} + Q_{4\ \text{after}}$$

$$Q_{4\ \text{after}} = \frac{10e}{2} = 5e$$

In conclusion, the final charge on sphere ❹ is 5e. Ans

2) The conservation of charges is applied to the contact of the spheres, just as in 1). Set the initial charge on sphere ❶ equal to Q_1, and the final charge on sphere ❹ equal to –2e, as follows:

Sphere ❶ is touched to sphere ❹

$$Q_{1\ \text{before}} + Q_{4\ \text{before}} = Q_{1\ \text{after}} + Q_{4\ \text{after}}$$

$$Q_1 + 0 = Q_{4\ \text{after}} + Q_{4\ \text{after}}$$

$$Q_{4\ \text{after}} = \frac{Q_1}{2}$$

Sphere ❷ is touched to sphere ❹

$$Q_{2\ \text{before}} + Q_{4\ \text{before}} = Q_{2\ \text{after}} + Q_{4\ \text{after}}$$

$$10e + \frac{Q_1}{2} = Q_{4\ \text{after}} + Q_{4\ \text{after}}$$

$$Q_{4\ \text{after}} = \frac{10e + \frac{Q_1}{2}}{2}$$

$$= 5e + \frac{Q_1}{4}$$

Sphere ❸ is touched to sphere ❹

$$Q_{3\ \text{before}} + Q_{4\ \text{before}} = Q_{3\ \text{after}} + Q_{4\ \text{after}}$$

$$(-5e) + \left(5e + \frac{Q_1}{4}\right) = Q_{4\ \text{after}} + Q_{4\ \text{after}}$$

$$Q_{4\ \text{after}} = \frac{\frac{Q_1}{4}}{2} = \frac{Q_1}{8}$$

Since the charge on sphere ❹ in the final step is equal to −2e, so

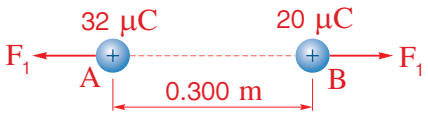
$$Q_{4\ \text{after}} = \frac{Q_1}{8} = -2e$$

$$Q_1 = 8 \times (-2e) = -16e$$

In conclusion, the initial charge on sphere ❶ is equal to −16e. Ans

Problem 17.20

Sphere A of charge 32 μC and sphere B of charge 20 μC are fixed 0.300 m apart and exert an electric force of magnitude F_1 on each other, as in Fig. (a). Electrically neutral sphere C is brought to touch sphere A, and then they are separated. Next, sphere C is touched to sphere B, and then they are separated. Finally, take sphere C far away at infinity. Now the magnitude of the electric force between spheres A and B has changed to F_2. Find the ratio $\frac{F_1}{F_2}$. Given the three spheres are the same.



(a)

Solution

Begin by calculating magnitude F_1 of the electric force in Fig. (a):

$$F_1 = k_0 \frac{|q_1 q_2|}{r^2}$$

$$F_1 = k_0 \frac{(32\ \mu C)(20\ \mu C)}{(0.300\ m)^2} \qquad \ldots (1)$$

Next, find the charges on spheres A and B after being touched to sphere C. Since all the spheres are identical, after touching, the charge on each sphere is the same.

Sphere A is touched to sphere C

$$Q_{A\ before} + Q_{C\ before} = Q_{A\ after} + Q_{C\ after}$$

$$32\ \mu C + 0 = Q_{A\ after} + Q_{A\ after}$$

$$Q_{A\ after} = Q_{C\ after} = \frac{32}{2}\ \mu C = 16\ \mu C$$

Sphere B is touched to sphere C

$$Q_{B\ before} + Q_{C\ before} = Q_{B\ after} + Q_{C\ after}$$

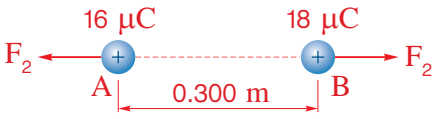
$$20\ \mu C + 16\ \mu C = Q_{B\ after} + Q_{B\ after}$$

$$Q_{B\ after} = \frac{36}{2}\ \mu C = 18\ \mu C$$

After sphere C was touched to spheres A and B, respectively, the charges on spheres A and B are shown in Fig. (b), and magnitude F_2 of the electric force is obtained by

$$F_2 = k_0 \frac{|q_1 q_2|}{r^2}$$

$$F_2 = k_0 \frac{(16\ \mu C)(18\ \mu C)}{(0.300\ m)^2} \qquad \text{... (2)}$$



(b)

Divide Eq. (1) by Eq. (2):

$$\frac{F_1}{F_2} = \frac{k_0 \dfrac{(32\ \mu C)(20\ \mu C)}{(0.300\ m)^2}}{k_0 \dfrac{(16\ \mu C)(18\ \mu C)}{(0.300\ m)^2}}$$

$$\frac{F_1}{F_2} = \frac{20}{9} \qquad \text{Ans}$$

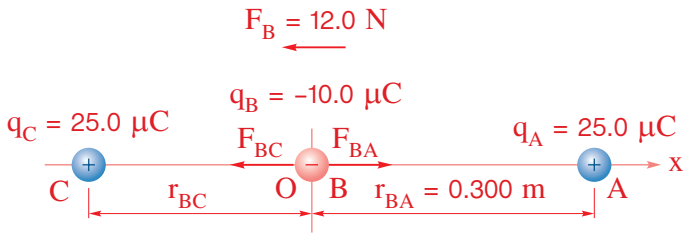
Problem 17.21

Charges A and B lie on the x-axis, as shown in Fig. (a). Find the position of charge C so that the resultant electric force on charge B has a magnitude of 12.0 N in the negative x-direction. Given $q_A = q_C = 25.0\ \mu C$ and $q_B = -10.0\ \mu C$.

(a)

Solution

When placing charge C, the resultant electric force on charge B is the vector sum of the electric forces on charge B due to charges A and C. Since charges A and B have opposite signs, the electric force on charge B due to charge A is attractive along the positive x-axis and of magnitude F_{BA}, as in Fig. (b).



(b)

For the resultant electric force on charge B pointing in the negative x-direction, the electric force on charge B due to charge C must be directed in the negative x-direction. Since charges B and C are of opposite signs, the electric force between these two charges is attractive. This causes charge C located on the left of charge B, far away at a distance r_{BC}, as shown in Fig (b). Given the magnitude of this force equal to F_{BC}.

Since the resultant electric force on charge B is of magnitude 12.0 N in the negative x-direction, magnitude F_{BC} is greater than magnitude F_{BA} by 12.0 N, i.e.

$$F_{BC} - F_{BA} = 12.0 \quad \text{... (1)}$$

Magnitudes F_{BA} and F_{BC} are given by

$$F_{BA} = k_0 \frac{|q_B q_A|}{r_{BA}^2} \quad \text{... (2)}$$

$$F_{BC} = k_0 \frac{|q_B q_C|}{r_{BC}^2} \quad \text{... (3)}$$

Substitute F_{BA} and F_{BC} from Eqs. (2) and (3) into Eq. (1), and then solve the equation for distance r_{BC} to find the position of charge C:

$$k_0 \frac{|q_B q_C|}{r_{BC}^2} - k_0 \frac{|q_B q_A|}{r_{BA}^2} = 12.0$$

$$k_0 \left(\frac{|q_B q_C|}{r_{BC}^2} - \frac{|q_B q_A|}{r_{BA}^2} \right) = 12.0$$

$$\frac{|q_B q_C|}{r_{BC}^2} = \frac{12.0}{k_0} + \frac{|q_B q_A|}{r_{BA}^2}$$

$$r_{BC} = \sqrt{\frac{|q_B q_C|}{\frac{12.0}{k_0} + \frac{|q_B q_A|}{r_{BA}^2}}}$$

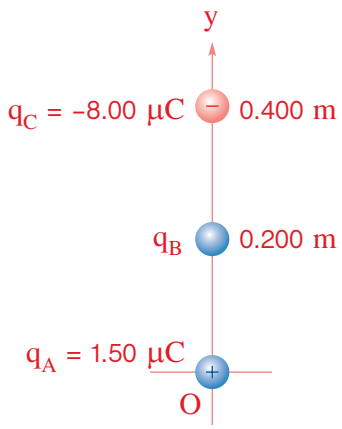
Substituting $q_A = q_C = 25.0\ \mu C = 25.0 \times 10^{-6}$ C, $q_B = -10.0\ \mu C = -10.0 \times 10^{-6}$ C, $r_{BA} = 0.300$ m, and $k_0 = 8.99 \times 10^9\ N \cdot m^2/C^2$ into the above expression gives

$$r_{BC} = 0.247 \text{ m}$$

The result shows that the position of charge C is on the left of the origin O by distance $r_{BC} = 0.247$ m; therefore, charge C is at $x = -0.247$ m. Ans

Problem 17.22

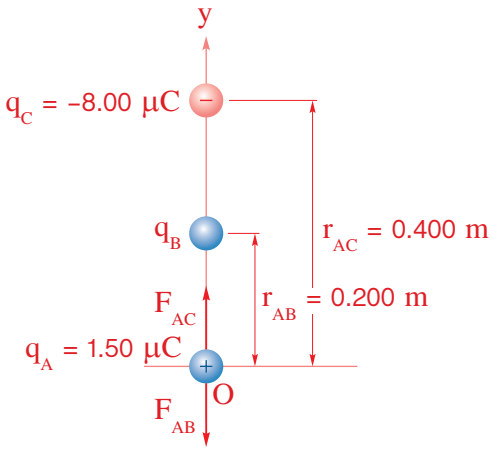
Charges A, B, and C lie on the y-axis, as shown in Fig. (a). What is charge q_B for the resultant electric force on charge q_A equal to zero?



(a)

Solution

The resultant electric force on charge q_A is the vector sum of the electric forces on charge q_A due to q_B and q_C. Since charges q_A and q_C have opposite signs, the electric force on charge q_A from charge q_C is attractive along the positive y-axis and of magnitude F_{AC}, as shown in Fig. (b).



(b)

For the resultant electric force on charge q_A equal to zero, the electric force on charge q_A due to charge q_B must be directed downward along the negative y-axis. When considering the position of charge q_A with respect to charge q_B, it is found that this force is repulsive, so charge q_B is positive, just as charge q_A. Given this force has a magnitude of F_{AB}.

Since the resultant electric force on charge q_A is equal to zero, magnitudes F_{AB} and F_{AC} must be the same:

$$F_{AB} = F_{AC} \qquad \text{... (1)}$$

Find magnitudes F_{AB} and F_{AC}, as follows:

$$F_{AB} = k_0 \frac{|q_A q_B|}{r_{AB}^2} \qquad \text{... (2)}$$

$$F_{AC} = k_0 \frac{|q_A q_C|}{r_{AC}^2} \qquad \text{... (3)}$$

Substitute F_{AB} and F_{AC} from Eqs. (2) and (3) into Eq. (1) to solve for q_B:

$$k_0 \frac{|q_A q_B|}{r_{AB}^2} = k_0 \frac{|q_A q_C|}{r_{AC}^2}$$

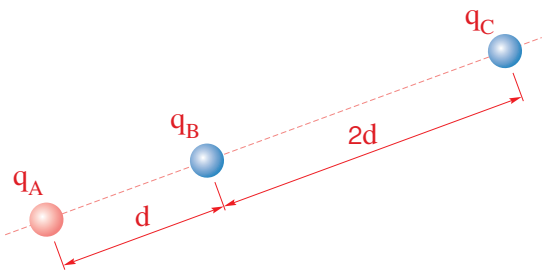
$$q_B = |q_C| \left(\frac{r_{AB}}{r_{AC}} \right)^2$$

$$= |-8.00\ \mu C| \times \left(\frac{0.200\ m}{0.400\ m} \right)^2$$

$$q_B = 2.00\ \mu C \qquad \text{Ans}$$

Problem 17.23

Charges q_A, q_B, and q_C are arranged along a line, as shown in Fig. (a). If the resultant electric force on charge q_A is equal to zero, find the ratio $\frac{q_B}{q_C}$.



(a)

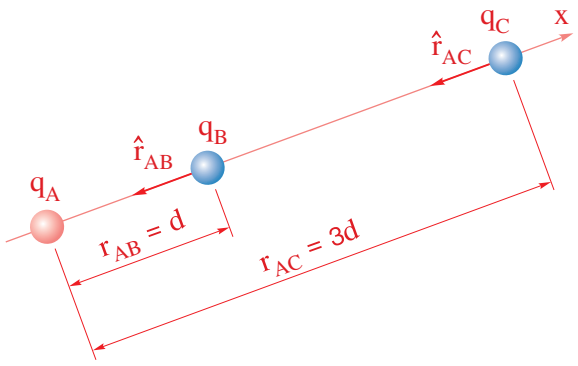
Solution

An essential condition for writing an equation to solve for the ratio $\frac{q_B}{q_A}$ is that "the resultant electric force on charge q_A is equal to zero." Since this resultant electric force is the vector sum of the electric forces on charge q_A due to charges q_B and q_C, therefore

$$\vec{F}_A = \vec{F}_{AB} + \vec{F}_{AC} = 0 \qquad \ldots (1)$$

However, the problem does not give any details of the three charges. So, we will find electric forces $\vec{F}_{AB}$ and $\vec{F}_{AC}$ in vector form to cover all cases of the electric forces, as follows:

Set the x–axis on the straight line, as in Fig. (b) to define easily the unit vector that locates charge q_A from other charges.



(b)

Electric force $\vec{F}_{AB}$ can be written in vector form as

$$\vec{F}_{AB} = k_0 \frac{q_A q_B}{r_{AB}^2} \hat{r}_{AB} \quad \text{... (2)}$$

In Fig. (b), $\hat{r}_{AB}$ is a unit vector for positioning charge q_A by an observer at charge q_B. We can see that $\hat{r}_{AB}$ is the unit vector directed along the negative x-axis; therefore, $\hat{r}_{AB} = -\vec{i}$. The figure also tells us that the distance between charges q_A and q_B is $r_{AB} = d$. Substituting these quantities into Eq. (2) leads to

$$\vec{F}_{AB} = k_0 \frac{q_A q_B}{d^2}(-\vec{i})$$

$$\vec{F}_{AB} = -k_0 \frac{q_A q_B}{d^2} \vec{i} \quad \text{... (3)}$$

Electric force $\vec{F}_{AC}$ can be written in vector form as

$$\vec{F}_{AC} = k_0 \frac{q_A q_C}{r_{AC}^2} \hat{r}_{AC} \quad \text{... (4)}$$

Similarly, $\hat{r}_{AC}$ is a unit vector that represents the position of charge q_A observed from charge q_C. In Fig. (b), $\hat{r}_{AC}$ is directed along the negative x-axis; therefore, $\hat{r}_{AC} = -\vec{i}$, and the distance between the charges is $r_{AC} = 3d$. Substitute the values of $\hat{r}_{AC}$ and r_{AC} into Eq. (4) to get

$$\vec{F}_{AC} = k_0 \frac{q_A q_C}{(3d)^2}(-\vec{i})$$

$$\vec{F}_{AC} = -\frac{k_0}{9} \frac{q_A q_C}{d^2} \vec{i} \quad \text{... (5)}$$

Substitute $\vec{F}_{AB}$ and $\vec{F}_{AC}$ from Eqs. (3) and (5) into Eq. (1) to solve for the ratio $\frac{q_B}{q_A}$:

$$\left(-k_0 \frac{q_A q_B}{d^2} \vec{i}\right) + \left(-\frac{k_0}{9} \frac{q_A q_C}{d^2} \vec{i}\right) = 0$$

$$-k_0 \frac{q_A q_B}{d^2} \vec{i} = \frac{k_0}{9} \frac{q_A q_C}{d^2} \vec{i}$$

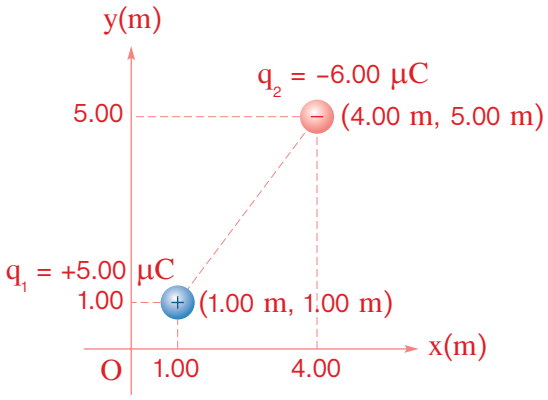
$$\frac{q_B}{q_C} = -\frac{1}{9} \quad \text{Ans}$$

Remarks

The result $\frac{q_B}{q_C} = -\frac{1}{9}$ means that charges q_B and q_C have opposite signs.

Problem 17.24

Charges $q_1 = +5.00\ \mu C$ and $q_2 = -6.00\ \mu C$ are placed in a rectangular coordinate system x–y at positions (1.00 m, 1.00 m) and (4.00 m, 5.00 m), respectively, as in Fig. (a). What is position (x, y) of charge $q_3 = +8.00\ \mu C$ making charge q_2 in equilibrium?



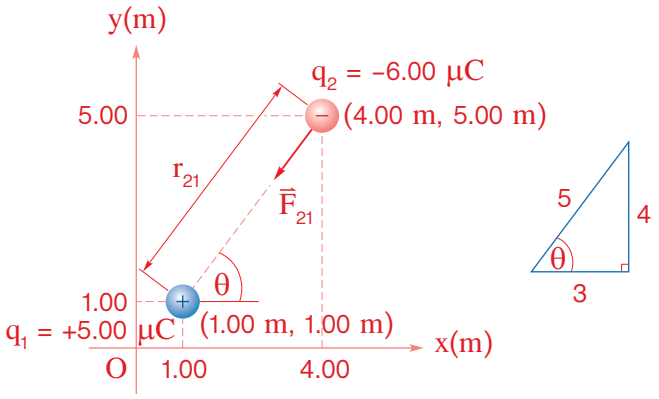
(a)

Solution

When placing charge q_3 in the x–y plane, two electric forces are exerted on charge q_2: electric forces $\vec{F}_{21}$ and $\vec{F}_{23}$ due to charges q_1 and q_3, respectively. So, charge q_2 will be in equilibrium by these two forces if they have the same magnitude and opposite direction.

We begin by finding electric force $\vec{F}_{21}$. Next, use it together with "the sum of electric forces $\vec{F}_{21}$ and $\vec{F}_{23}$ equals zero" to calculate $\vec{F}_{23}$. Finally, we can apply the known electric force $\vec{F}_{23}$ to find the position (x, y) of charge q_3, as follows:

Figure (b) shows electric force $\vec{F}_{21}$ on charge q_2 exerted by q_1. This force is attractive along the line connecting the two charges. The line makes an angle θ above the horizontal, indicated by a right triangle of sides 3, 4, and 5, on the right of the figure.



(b)

Find the magnitude of electric force $\vec{F}_{21}$:

$$F_{21} = k_0 \frac{|q_1 q_2|}{r_{21}^2} \quad \text{... (1)}$$

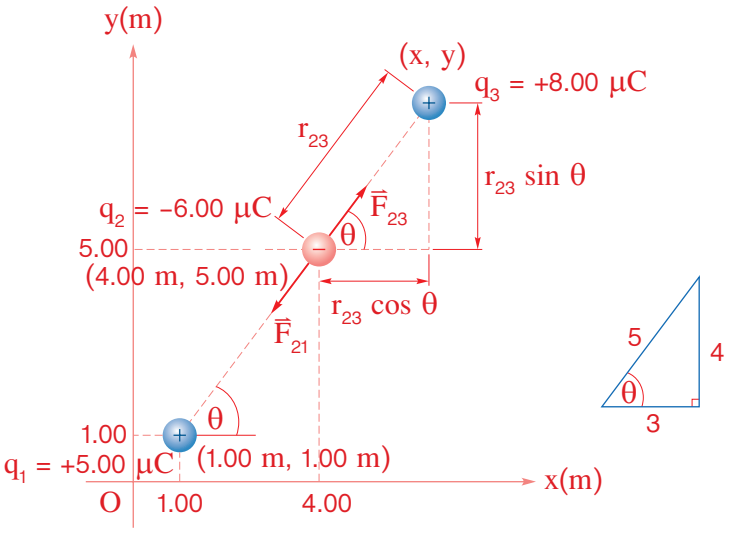
Distance r_{21} between charges q_1 and q_2 is given by the Pythagorean Theorem and their coordinates:

$$r_{21} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4.00 - 1.00)^2 + (5.00 - 1.00)^2}$$

$$r_{21} = 5.00 \text{ m} \quad \text{... (2)}$$

Now we know the magnitude and direction of force $\vec{F}_{21}$, then consider electric force $\vec{F}_{23}$, as shown in Fig. (c).



(c)

Since charge q_2 will be in equilibrium if the sum of forces $\vec{F}_{23}$ and $\vec{F}_{21}$ is equal to zero. This causes force $\vec{F}_{23}$ making an angle θ above the horizontal, opposite to the direction of force $\vec{F}_{21}$, as shown in Fig. (c). Because of the opposite signs of charges q_2 and q_3, force $\vec{F}_{23}$ is attractive, and it indicates that the position of charge q_3 is on the top right of charge q_2, as located in Fig. (c).

Set charge q_3 is at position (x, y), away from charge q_2 by distance r_{23}. So, the coordinates x and y are given by

$$x = 4.00\ \text{m} + r_{23} \cos \theta \qquad \ldots (3)$$

$$y = 5.00\ \text{m} + r_{23} \sin \theta \qquad \ldots (4)$$

From Eqs. (3) and (4), we see that if knowing distance r_{23}, the required coordinates x and y would be obtained. Distance r_{23} is given by the expression for the equal magnitudes of force $\vec{F}_{21}$ and $\vec{F}_{23}$, as follows:

$$F_{21} = F_{23}$$

$$k_0 \frac{|q_1 q_2|}{r_{21}^2} = k_0 \frac{|q_2 q_3|}{r_{23}^2}$$

$$r_{23} = \sqrt{\frac{|q_3|}{|q_1|}}\, r_{21}$$

$$= \sqrt{\frac{|+8.00\ \mu C|}{|+5.00\ \mu C|}} \times 5.00\ m$$

$$r_{23} = 6.32\ m$$

Substituting $r_{23} = 6.32$ m into Eqs. (3) and (4) and using $\sin\theta = \frac{4}{5}$ and $\cos\theta = \frac{3}{5}$ gives us

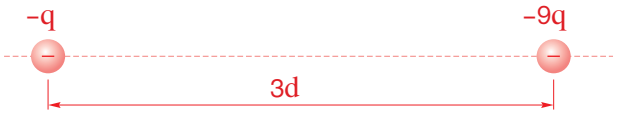
$$x = 4.00 + 6.32 \times \frac{3}{5} = 7.79\ m$$

$$y = 5.00 + 6.32 \times \frac{4}{5} = 10.1\ m$$

It can be concluded that charge q_3 is at position (7.79 m, 10.1 m). Ans

Problem 17.25

Charges –q and –9q are fixed a distance 3d apart, as shown in Fig. (a). Find charge Q_x and its position that causes each of the charges in equilibrium by the electric forces from the other two charges.



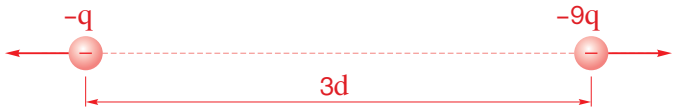
(a)

Solution

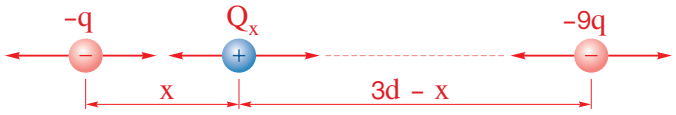
When placing charge Q_x in a region of charges –q and –9q, two electric forces are exerted on each charge due to the other two charges. Therefore, to get the resultant electric force on each charge equal to zero and the charge in equilibrium, the two electric forces on each charge must be equal in magnitude and opposite in direction. From this, all charges must be aligned along the same line to get the

opposite direction of the two forces on each charge.

Charges –q and –9q have opposite signs, the electric forces between them are therefore repulsive, as shown in Fig. (b). When lying charge Q_x, it must exert an additional force on each of the charges –q and –9q in the opposite direction of the forces in Fig. (b), as shown in Fig. (c). From this situation, charge Q_x must be placed on the straight line between charges –q and –9q, and is positive, as in Fig. (c).



(b)



(c)

You should try to place charge Q_x on the left of charge –q, or on the right of charge –9q, and change its sign to be both positive and negative. It will find that the two electric forces on each charge cannot be of opposite direction simultaneously, that is, positive charge Q_x and its position in Fig. (c) are the single case in which all charges are in equilibrium.

Given charge Q_x placed away from charge –q a distance x, as shown in Fig. (c), then determine the resultant electric force on each charge. Set a positive direction to the right, we get

charge Q_x

$$\vec{F}_{Qx} = \left(+k_0 \frac{|Q_x \times (-9q)|}{(3d - x)^2} \right) + \left(-k_0 \frac{|Q_x \times (-q)|}{x^2} \right)$$

$$\vec{F}_{Qx} = \frac{9k_0 Q_x q}{(3d - x)^2} - \frac{k_0 Q_x q}{x^2} \qquad \text{... (1)}$$

charge –q

$$\vec{F}_{-q} = \left(+k_0 \frac{|(-q) \times Q_x|}{x^2} \right) + \left(-k_0 \frac{|(-q) \times (-9q)|}{(3d)^2} \right)$$

$$\vec{F}_{-q} = \frac{k_0 Q_x q}{x^2} - \frac{9k_0 q^2}{(3d)^2} \quad \text{... (2)}$$

charge –9q

$$\vec{F}_{-9q} = \left(-k_0 \frac{|(-9q) \times Q_x|}{(3d - x)^2} \right) + \left(+k_0 \frac{|(-9q) \times (-q)|}{(3d)^2} \right)$$

$$\vec{F}_{-9q} = \frac{-9k_0 Q_x q}{(3d - x)^2} + \frac{9k_0 q^2}{(3d)^2} \quad \text{... (3)}$$

Since we need the three charges in equilibrium simultaneously, set the resultant electric forces in Eqs. (1) through (3) equal to zero, as follows:

From Eq. (1),

$$\vec{F}_{Qx} = 0 = \frac{9k_0 Q_x q}{(3d - x)^2} - \frac{k_0 Q_x q}{x^2}$$

$$9 = \left(\frac{3d - x}{x} \right)^2$$

$$3 = \frac{3d - x}{x}$$

$$4x = 3d$$

$$x = \frac{3d}{4}$$

From Eq. (2),

$$\vec{F}_{-q} = 0 = \frac{k_0 Q_x q}{x^2} - \frac{9k_0 q^2}{(3d)^2}$$

$$Q_x = 9q\left(\frac{x}{3d}\right)^2$$

$$= 9q\left(\frac{\frac{3d}{4}}{3d}\right)^2 \quad , \; x = \frac{3d}{4}$$

$$Q_x = \frac{9}{16}q$$

In Eq. (1), as $\vec{F}_{Qx} = 0$, we get

$$\frac{9k_0Q_xq}{(3d - x)^2} = \frac{k_0Q_xq}{x^2} \qquad \ldots (4)$$

In Eq. (2), as $\vec{F}_{-q} = 0$, we get

$$\frac{9k_0q^2}{(3d)^2} = \frac{k_0Q_xq}{x^2} \qquad \ldots (5)$$

Substituting the quantities from Eqs. (4) and (5) into Eq. (3) leads to $\vec{F}_{-9q} = 0$, as the following:

$$\vec{F}_{-9q} = -\frac{9k_0Q_xq}{(3d - x)^2} + \frac{9k_0q^2}{(3d)^2}$$

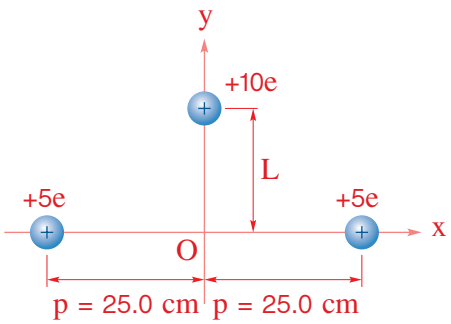
$$= -\left(\frac{k_0Q_xq}{x^2}\right) + \frac{k_0Q_xq}{x^2}$$

$$\vec{F}_{-9q} = 0$$

In conclusion, when charge Q_x is equal to $\frac{9}{16}q$ and located on the right of charge –q, a distance $\frac{3d}{4}$ apart, all three charges are in equilibrium simultaneously. Ans

Problem 17.26

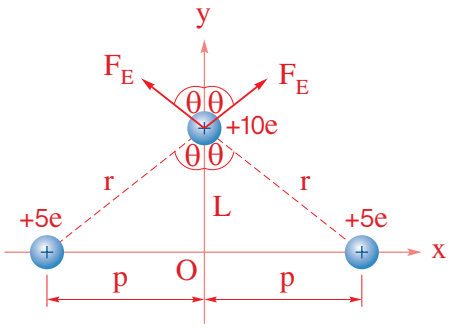
Two equal charges of +5e are located on the x-axis, each a distance 25.0 cm from the origin O, as shown in Fig. (a). Another charge of +10e is placed on the positive y-axis, a distance L from the origin O. What is the distance L to maximize the resultant electric force on the charge +10e? Also, find the magnitude of the maximum force.



(a)

Solution

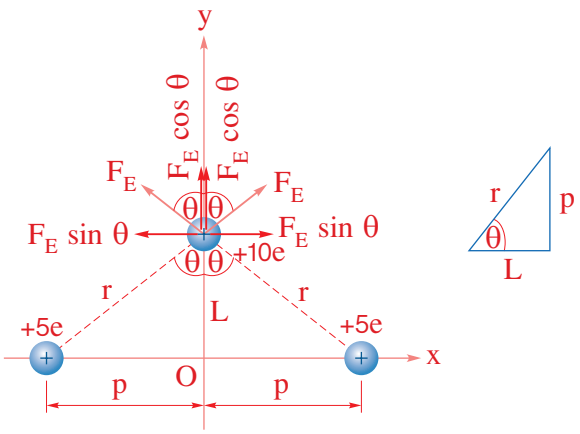
The resultant electric force on charge +10e is the vector sum of the electric forces due to both charges +5e exerted on charge +10e. Since both charges +5e are equidistant from charge +10e at distance r, as shown in Fig. (b), the electric forces from these two charges on charge +10e is repulsive and of the same magnitude. Set this magnitude as F_E, as in Fig. (b).



(b)

Moreover, because charge +10e is on the positive y-axis, which is the perpendicular bisector to the line connecting charges +5e, each of the electric forces makes an equal angle θ with the positive y-axis. We will use these data to determine the resultant electric force on charge +10e.

The resultant electric force is the sum of the two electric forces with equal magnitude F_E. Our procedures are: first, resolve each electric force into its components along the x- and y-axes, as shown in Fig. (c), then the components on the x-axis consist of two electric forces with the same magnitude $F_E \sin \theta$ and opposite direction, so the vector sum of all the x-components is zero.



(c)

On the y-axis, there are two components that have equal magnitude $F_E \cos \theta$ and are pointed in the same positive y-direction. Therefore, the resultant component of the electric force along the y-axis is also directed along the positive y-axis and of magnitude $2F_E \cos \theta$.

Since the resultant x-component equals zero, the resultant electric force is equal to the resultant y-component. The magnitude of the resultant electric force can be determined, as follows:

$$\sum F = \sum F_y = 2F_E \cos \theta \qquad \text{... (1)}$$

Magnitude F_E of the electric force can be calculated by

$$F_E = k_0 \frac{|q_1 q_2|}{r^2}$$

$$= k_0 \frac{|(+5e)(+10e)|}{r^2}$$

$$F_E = \frac{50 k_0 e^2}{r^2} \quad \text{... (2)}$$

where distance r is given by applying the Pythagorean Theorem on the right triangle in Fig. (c):

$$r^2 = p^2 + L^2$$

$$r = (p^2 + L^2)^{\frac{1}{2}} \quad \text{... (3)}$$

and the value of cos θ is found on the same right triangle:

$$\cos \theta = \frac{L}{r} \quad \text{... (4)}$$

Substitute F_E, r, and cos θ from Eqs. (2) through (4) into Eq. (1) to get the magnitude of the resultant electric force:

$$\sum F = 2F_E \cos \theta$$

$$= 2\left(\frac{50 k_0 e^2}{r^2}\right)\left(\frac{L}{r}\right)$$

$$= 100 k_0 \frac{e^2 L}{r^3}$$

$$= \frac{100 k_0 e^2 L}{(p^2 + L^2)^{\frac{3}{2}}} \quad \text{... (5)}$$

Next, do the derivative of the above expression with respect to L, then set it equal to zero, and solve for L, as follows:

$$\frac{d\sum F}{dL} = \frac{100k_0e^2(p^2 + L^2)^{\frac{3}{2}} - 100k_0e^2L\left(\frac{3}{2}\right)(p^2 + L^2)^{\frac{1}{2}}(2L)}{\left((p^2 + L^2)^{\frac{3}{2}}\right)^2}$$

$$= \frac{100k_0e^2(p^2 + L^2)^{\frac{3}{2}} - 300k_0e^2L^2(p^2 + L^2)^{\frac{1}{2}}}{(p^2 + L^2)^3}$$

Set $\frac{d\sum F}{dL} = 0$, we get

$$100k_0e^2(p^2 + L^2)^{\frac{3}{2}} - 300k_0e^2L^2(p^2 + L^2)^{\frac{1}{2}} = 0$$

$$(p^2 + L^2)^{\frac{3}{2}} = 3L^2(p^2 + L^2)^{\frac{1}{2}}$$

$$p^2 + L^2 = 3L^2$$

$$2L^2 - p^2 = 0$$

$$(\sqrt{2}\,L - p)(\sqrt{2}\,L + p) = 0$$

Therefore,

$$\sqrt{2}\,L - p = 0$$

$$L = \frac{p}{\sqrt{2}}$$

or,

$$\sqrt{2}\,L + p = 0$$

$$L = -\frac{p}{\sqrt{2}}$$

Because distance L must be a positive number,

$$L = \frac{p}{\sqrt{2}} = \frac{25.0\ \text{cm}}{\sqrt{2}} = 17.7\ \text{cm}$$

$$L = 0.177\ \text{m}$$

Substitute L = 0.177 m into Eq. (5) to get the maximum magnitude of the resultant electric force:

$$\Sigma F_{max} = \frac{100k_0e^2L}{(p^2 + L^2)^{\frac{3}{2}}}, \quad L = 0.177 \text{ m}$$

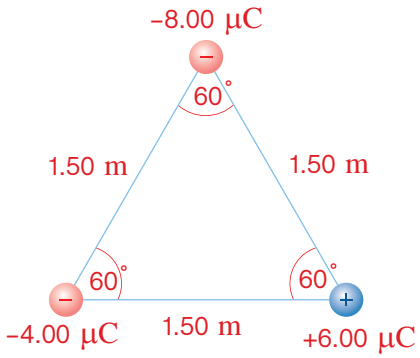
$$= \frac{100 \times 8.99 \times 10^9 \times (1.60 \times 10^{-19})^2 \times 0.177}{(0.250^2 + 0.177^2)^{\frac{3}{2}}}$$

$$\Sigma F_{max} = 1.42 \times 10^{-25} \text{ N}$$

It can be concluded that the resultant electric force on charge +10e has a maximum magnitude of 1.42×10^{-25} N when placing charge +10e a distance 17.7 cm or 0.177 m away from the origin O. Ans

Problem 17.27

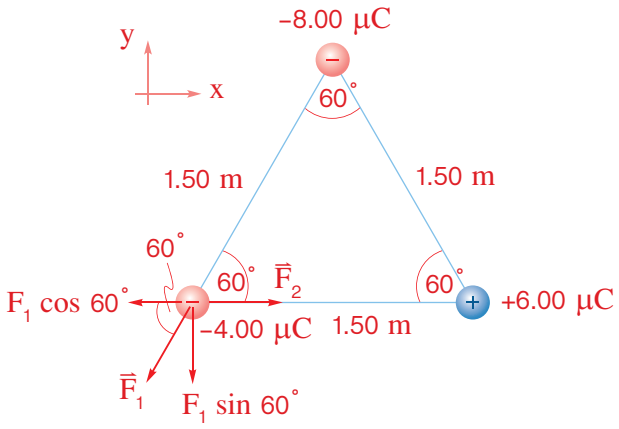
Three point charges are placed at the corners of an equilateral triangle of side 1.50 m, as shown in Fig. (a). What is the resultant electric force on charge −4.00 μC?



(a)

Solution

In Fig. (b), the resultant electric force on charge −4.00 μC is the vector sum of forces $\vec{F}_1$ and $\vec{F}_2$ on charge −4.00 μC exerted by charges −8.00 μC and +6.00 μC, respectively.



(b)

Forces $\vec{F}_1$ and $\vec{F}_2$ are along the lines connecting the charges, which are the triangle's sides. Force $\vec{F}_1$ is repulsive because charges −4.00 μC and −8.00 μC have the same sign; force $\vec{F}_2$ is attractive because of the opposite signs of charges −4.00 μC and +6.00 μC.

Since each internal angle of an equilateral triangle is 60°, the directions of the forces $\vec{F}_1$ and $\vec{F}_2$ are known, as shown in Fig. (b). Next, we would find the magnitude of the two forces, then add them vectorially to get the desired resultant electric force, as follows:

$$F_E = k_0 \frac{|q_1 q_2|}{r^2}$$

$$F_1 = 8.99 \times 10^9 \frac{\left|(-4.00 \times 10^{-6})(-8.00 \times 10^{-6})\right|}{1.50^2}$$

$$F_1 = 0.127858 \text{ N}$$

$$F_2 = 8.99 \times 10^9 \frac{\left|(-4.00 \times 10^{-6})(+6.00 \times 10^{-6})\right|}{1.50^2}$$

$$F_2 = 0.095893 \text{ N}$$

Add forces $\vec{F}_1$ and $\vec{F}_2$ by resolving them into their x- and y-components, as in Fig. (b), then find the resultant x- and y-components:

$$\sum \vec{F}_x = (+F_2) + (-F_1 \cos 60^\circ)$$

$$= +0.095893 + (-0.127858 \cos 60^\circ)$$

$$\sum \vec{F}_x = 0.031964 \text{ N}$$

$$\sum \vec{F}_y = -F_1 \sin 60^\circ$$

$$= -0.127858 \sin 60^\circ$$

$$\sum \vec{F}_y = -0.110728 \text{ N}$$

Now we know the x- and y-components of the resultant electric force, therefore, the magnitude of the resultant electric force is

$$\sum F = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2}$$

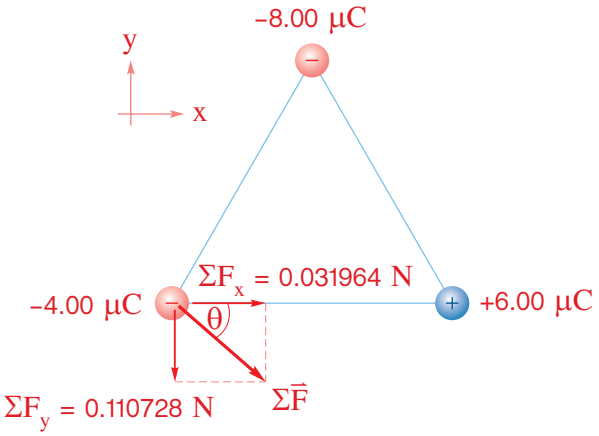
$$= \sqrt{0.031964^2 + 0.110728^2}$$

$$\sum F = 1.15 \times 10^{-1} \text{ N}$$

The direction of the resultant electric force is indicated by an angle θ in Fig. (c):

$$\tan \theta = \frac{0.110728}{0.031964}$$

$$\theta = 73.90^\circ$$

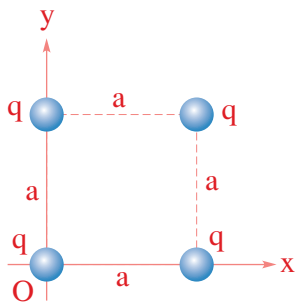


(c)

In conclusion, the resultant electric force on charge -4.00 μC is 1.15×10^{-1} N, directed at an angle of 73.90° below the positive x-axis. Ans

Problem 17.28

Four point charges q are fixed at the corners of a square of side a, as shown in Fig. (a). Find the magnitude and direction of the resultant electric force on charge q at the origin O and evaluate that force for q = 6.00 μC and a = 0.150 m.

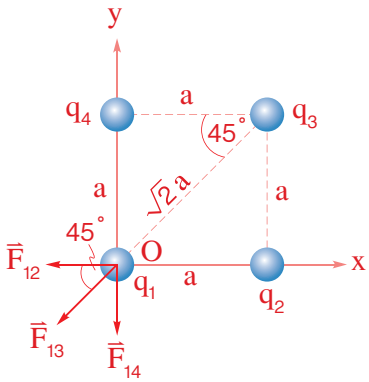


(a)

Solution

We represent each charge q as q_1, q_2, q_3, and q_4, where $q_1 = q_2 = q_3 = q_4 = q$, as in Fig. (b). From the figure, the required force is the resultant electric force, $\vec{F}_1$, on charge q_1, which is the vector sum of the electric forces due to charges q_2, q_3, and q_4 on charge q_1, as follows:

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} \qquad \ldots (1)$$



(b)

In Fig. (b), electric forces $\vec{F}_{12}, \vec{F}_{13}$, and $\vec{F}_{14}$ are repulsive because all the charges have the same sign. So, force $\vec{F}_{12}$ is directed along the negative x-axis, force $\vec{F}_{14}$

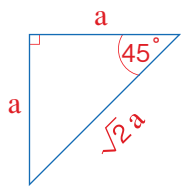
along the negative y-axis, and force $\vec{F}_{13}$ along the diagonal, making a 45° angle below the negative x-axis.

Since charge q_1 is equidistant by a distance a from charges q_2 and q_4, the magnitudes of electric forces $\vec{F}_{12}$ and $\vec{F}_{14}$ are the same and given by

$$F_{12} = F_{14} = k_0 \frac{qq}{a^2} = k_0 \frac{q^2}{a^2} \quad \text{... (2)}$$

However, the magnitude of force $\vec{F}_{13}$ is not equal to that of forces $\vec{F}_{12}$ and $\vec{F}_{14}$ because the distance between the charges q_1 and q_3 is $\sqrt{2}a$, not a, according to the right triangle in Fig. (c). So, the magnitude of force $\vec{F}_{13}$ is

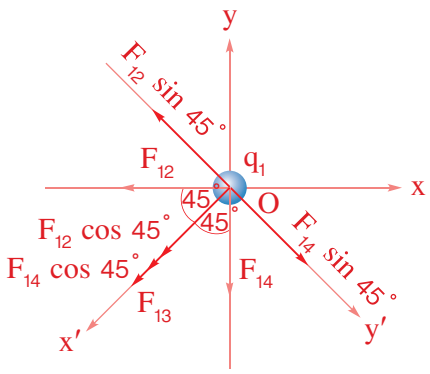
$$F_{13} = k_0 \frac{qq}{(\sqrt{2}a)^2} = k_0 \frac{q^2}{2a^2} \quad \text{... (3)}$$



(c)

When the magnitude and direction of forces $\vec{F}_{12}, \vec{F}_{13}$, and $\vec{F}_{14}$ are known, the resultant force $\vec{F}_1$ can be obtained. We generally use the method of resolving forces $\vec{F}_{12}, \vec{F}_{13}$ and $\vec{F}_{14}$ into their x- and y-components, then sum all the x- and y-components separately, and add the resultant components on the x- and y-axes to get the resultant electric force. However, since forces $\vec{F}_{12}$ and $\vec{F}_{14}$ have the same magnitude, we would find the sum of the components along the x′- and y′-axes instead of the x- and y-axes, as shown in Fig. (d).

Figure (d) shows another rectangular coordinate system x′-y′, where the x′-axis is in the same direction as force $\vec{F}_{13}$ and the y′-axis is perpendicular to the x′-axis. By such the coordinate system, no need to resolve $\vec{F}_{13}$. Because forces $\vec{F}_{12}$ and $\vec{F}_{14}$ make an equal angle of 45° with the x′-axis, after resolving the forces into their x′- and y′-components, we therefore find that, on the y′-axis, there are two forces of equal magnitude, F_{12} sin 45° and F_{14} sin 45° ($F_{12} = F_{14}$), in the opposite direction, so these two forces add to zero. In conclusion, the resultant electric force is just the sum of all the x′-components.



(d)

On the x′–axis, there are three force components in the same positive x′–direction, and of magnitudes F_{13}, F_{12} cos 45°, and F_{14} cos 45°. Therefore, the resultant electric force is also directed along the positive x′–axis and has a magnitude of

$$F_1 = F_{13} + F_{12}\cos 45^\circ + F_{14}\cos 45^\circ$$

$$= F_{13} + 2F_{12}\cos 45^\circ \quad , \quad (F_{12} = F_{14})$$

$$= \frac{k_0 q^2}{2a^2} + 2\left(\frac{k_0 q^2}{a^2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{k_0 q^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

$$F_1 = \frac{k_0 q^2}{2a^2}\left(1 + 2\sqrt{2}\right) \qquad \text{... (4)}$$

In conclusion, the resultant electric force on charge q at the origin O has magnitude $\frac{k_0 q^2}{2a^2}\left(1 + 2\sqrt{2}\right)$, directed a 45° angle below the negative x–axis. Ans

Substitute q = 6.00 μC = 6.00×10^{-6} C and a = 0.150 m into Eq. (4):

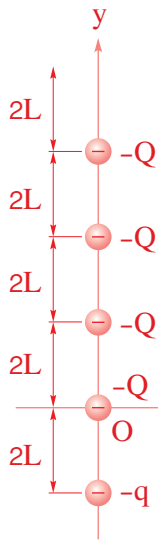
$$F_1 = \frac{8.99 \times 10^9 \times (6.00 \times 10^{-6})^2}{2 \times 0.150^2}\left(1 + 2\sqrt{2}\right)$$

$$F_1 = 27.5 \text{ N}$$

Therefore, the resultant electric force on charge q at the origin O has a magnitude of 27.5 N, directed a 45° angle below the negative x–axis. Ans

Problem 17.29

Infinite point charges –Q are aligned on the positive y–axis, starting at the origin O to infinitely positive y–value, each placed a distance 2L apart, as shown in Fig. (a). If charge –q is fixed at position y = –2L on the y–axis, what is the resultant electric force acting on charge –q due to all the charges –Q?



(a)

Solution

The resultant electric force, $\vec{F}_E$, on the charge –q is the vector sum of the electric forces due to each of charges –Q. Since charges –q and –Q have the same sign, all the electric forces between them are repulsive, and the resultant electric force $\vec{F}_E$ is also repulsive along the negative y–axis, that is

$$\vec{F}_E = -F_E \vec{j} \qquad \text{... (1)}$$

Magnitude F_E of the resultant electric force is the sum of all the electric forces' magnitudes between charge –q and each of charge –Q. The first charge –Q is at y = 0, a distance 2L from charge –q. The distance between the next charge –Q and charge –q is increased by 2L as 4L, and the other charges –Q is like this: the distances are 6L, 8L, 10L, ..., as shown in Fig. (b). Therefore, the magnitude of the resultant electric force is

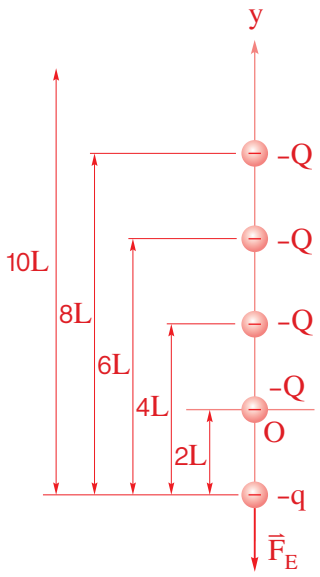
$$F_E = k_0 \frac{qQ}{(2L)^2} + k_0 \frac{qQ}{(4L)^2} + k_0 \frac{qQ}{(6L)^2} + k_0 \frac{qQ}{(8L)^2} + \cdots$$

$$= k_0 \frac{qQ}{L^2}\left(\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \frac{1}{8^2} + \ldots\right)$$

$$= k_0 \frac{qQ}{L^2}\left(\frac{1}{(2\times 1)^2} + \frac{1}{(2\times 2)^2} + \frac{1}{(2\times 3)^2} + \frac{1}{(2\times 4)^2} + \ldots\right)$$

$$= k_0 \frac{qQ}{L^2} \times \frac{1}{2^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\right)$$

$$F_E = k_0 \frac{qQ}{L^2} \times \frac{1}{2^2}\left(\sum_{n=1}^{\infty} \frac{1}{n^2}\right) \qquad \ldots (2)$$



(b)

The sum of the infinite series in Eq. (2) is

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

Therefore,

$$F_E = k_0 \frac{qQ}{L^2} \times \frac{1}{2^2}\left(\frac{\pi^2}{6}\right)$$

$$F_E = \frac{k_0 \pi^2}{24} \frac{qQ}{L^2}$$

Finally, substitute magnitude F_E that we have just found into Eq. (1):

$$\vec{F}_E = -\frac{k_0 \pi^2}{24} \frac{qQ}{L^2} \vec{j}$$ Ans

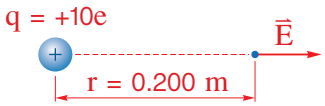
"Electric Field due to Point Charges"

Problem 17.30

What are the magnitude and direction of an electric field due to charge +10e at a point to the right, a distance 0.200 m far away?

Solution

An electric field due to a positive point charge is directed radially outward from the charge. Therefore, electric field $\vec{E}$ due to charge +10e at a point on the right is directed to the right, as shown in the figure.



Magnitude E of the electric field is

$$E = k_0 \frac{|q|}{r^2}$$

$$= 8.99 \times 10^9 \times \frac{\left|10 \times 1.60 \times 10^{-19}\right|}{0.200^2}$$

$$E = 3.60 \times 10^{-7} \text{ N/C}$$

It can be concluded that the electric field has a magnitude of 3.60×10^{-7} N/C directed to the right. Ans

Problem 17.31

An electric field due to a charge has a magnitude of 6.40 N/C at a point 30.0 cm far away from the charge. Find the magnitude of the charge.

Solution

The magnitude |q| of the charge can be obtained by the expression for magnitude E of an electric field:

$$E = k_0 \frac{|q|}{r^2}$$

$$|q| = \frac{Er^2}{k_0} \quad \text{... (1)}$$

Substitute E = 6.40 N/C, r = 30.0 cm = 0.300 m, and $k_0 = 8.99 \times 10^9$ N·m^2/C^2 into Eq. (1):

$$|q| = \frac{6.40 \times 0.300^2}{8.99 \times 10^9}$$

$$|q| = 6.41 \times 10^{-11} \text{ C} \quad \text{Ans}$$

Remarks

If the type of the charge is required, more information about the electric field's direction must be given.

Problem 17.32

At what distance from charge 6.50 μC does the electric field due to the charge have a magnitude of 36.0 N/C?

Solution

The problem gives q = 6.50 μC = 6.50×10^{-6} C and E = 36.0 N/C, so the distance r is obtained by the expression for an electric field's magnitude, as follows:

$$E = k_0 \frac{|q|}{r^2}$$

$$r = \sqrt{\frac{k_0 |q|}{E}}$$

$$= \sqrt{\frac{8.99 \times 10^9 \times |6.50 \times 10^{-6}|}{36.0}}$$

$$r = 40.3 \text{ m} \quad \text{Ans}$$

Problem 17.33

An electron is in a uniform electric field. If the electric force exerted on the electron has a magnitude of 3.20×10^{-16} N eastward, find the magnitude and direction of the electric field.

Solution

If a charge and the electric force on it are known, the electric field is given by

$$\text{electric field} = \frac{\text{electric force on the charge}}{\text{charge}}$$

Substitute the electric force equal to 3.20×10^{-16} N eastward and the charge equal to -1.60×10^{-19} C (an electron's charge) into the above expression to find the required electric field:

$$\text{electric field} = \frac{3.20 \times 10^{-16} \text{ N eastward}}{-1.60 \times 10^{-19} \text{ C}}$$

$$= -2.00 \times 10^{3} \text{ N/C eastward}$$

$$\text{electric field} = 2.00 \times 10^{3} \text{ N/C westward} \quad \text{Ans}$$

Problem 17.34

A proton is in an electric field of magnitude 1.50×10^{3} N/C in the upward direction. What is the electric force exerted on the proton?

Solution

If a charge and an electric field are known, the electric force on the charge is given by

$$\text{electric force} = \text{charge} \times \text{electric field} \quad \ldots (1)$$

Substituting the charge equal to $+1.60 \times 10^{-19}$ C (a proton's charge) and the electric field equal to 1.50×10^{3} N/C upward into Eq. (1), we get

$$\text{electric force} = (+1.60 \times 10^{-19}\ \text{C}) \times (1.50 \times 10^{3}\ \text{N/C upward})$$

$$\text{electric force} = 2.40 \times 10^{-16}\ \text{N upward} \qquad \text{Ans}$$

Problem 17.35

Charge +4.00 μC is in an electric field. If the electric force exerted on the charge equals $6.00 \times 10^{-3}\,\vec{i} - 9.00 \times 10^{-3}\,\vec{j}$ N, find the electric field at the charge position.

Solution

When charge q and electric force $\vec{F}_E$ are known, we can find electric field $\vec{E}$, as follows:

$$\vec{E} = \frac{\vec{F}_E}{q} \qquad \ldots (1)$$

Substituting $q = +4.00\ \mu\text{C} = +4.00 \times 10^{-6}$ C and $\vec{F}_E = 6.00 \times 10^{-3}\,\vec{i} - 9.00 \times 10^{-3}\,\vec{j}$ N into Eq. (1) leads to

$$\vec{E} = \frac{6.00 \times 10^{-3}\,\vec{i} - 9.00 \times 10^{-3}\,\vec{j}\ \text{N}}{+4.00 \times 10^{-6}\ \text{C}}$$

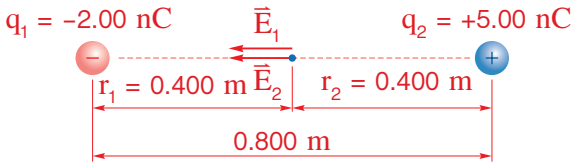
$$\vec{E} = 1.50 \times 10^{3}\,\vec{i} - 2.25 \times 10^{3}\,\vec{j}\ \text{N/C} \qquad \text{Ans}$$

Problem 17.36

Charges −2.00 nC and +5.00 nC are 0.800 m apart. Find the electric field at the midpoint between the charges.

Solution

Electric field $\vec{E}$ at the midpoint between charges $q_1 = -2.00$ nC and $q_2 = +5.00$ nC is the vector sum of electric fields $\vec{E}_1$ and $\vec{E}_2$ due to both charges, as shown in the figure.



Basically, an electric field is directed outward from a positive charge, and towards a negative charge. Hence, electric field $\vec{E}_1$ from charge q_1 is pointed to the left towards charge q_1; and electric field $\vec{E}_2$ is pointed to the left outward from charge q_2. Since electric fields $\vec{E}_1$ and $\vec{E}_2$ both are pointed to the left, the resultant electric field $\vec{E}$ is also pointed to the left.

The magnitude of the resultant electric field is the sum of the magnitudes of electric fields $\vec{E}_1$ and $\vec{E}_2$, as follows:

$$E = E_1 + E_2 \qquad \text{... (1)}$$

where $E_1 = k_0 \dfrac{|q_1|}{r_1^2}$ and $E_2 = k_0 \dfrac{|q_2|}{r_2^2}$

Therefore,

$$E = k_0 \frac{|q_1|}{r_1^2} + k_0 \frac{|q_2|}{r_2^2}$$

$$= \frac{k_0}{r^2}\left(|q_1| + |q_2|\right), \quad r_1 = r_2 = r = 0.400 \text{ m}$$

$$= \frac{8.99 \times 10^9}{0.400^2}\left(\left|-2.00 \times 10^{-9}\right| + \left|+5.00 \times 10^{-9}\right|\right)$$

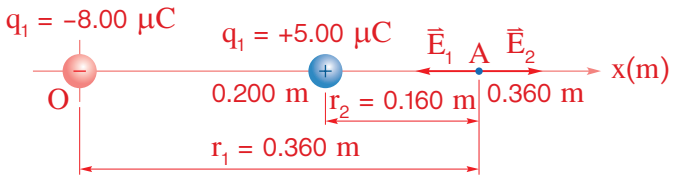
$$E = 393 \text{ N/C}$$

In conclusion, the electric field at the midpoint between the charges has a magnitude of 393 N/C towards the negative charge. Ans

Problem 17.37

Charge −8.00 μC is at the origin O, and charge +5.00 μC is on the x-axis at x = 0.200 m. Find the resultant electric field on the x-axis at x = 0.360 m.

Solution



From the figure, the resultant electric field at x = 0.360 m on the x-axis, point A, is the vector sum of electric fields $\vec{E}_1$ and $\vec{E}_2$ due to charges q_1 and q_2, respectively, therefore

$$\vec{E}_A = \vec{E}_1 + \vec{E}_2 \qquad \text{... (1)}$$

Electric field $\vec{E}_1$ is directed along the negative x-axis towards charge q_1 since q_1 is a negative charge; electric field $\vec{E}_2$ is directed along the positive x-axis outward from positive charge q_2, so we can write each of the electric fields as

$$\vec{E}_1 = k_0 \frac{|q_1|}{r_1^2}(-\vec{i})$$

$$\vec{E}_2 = k_0 \frac{|q_2|}{r_2^2}(+\vec{i})$$

Substituting electric fields $\vec{E}_1$ and $\vec{E}_2$ into Eq. (1), we have

$$\vec{E}_A = \left(-k_0 \frac{|q_1|}{r_1^2}\vec{i}\right) + \left(k_0 \frac{|q_2|}{r_2^2}\vec{i}\right)$$

$$= k_0 \left(-\frac{|q_1|}{r_1^2} + \frac{|q_2|}{r_2^2}\right)\vec{i}$$

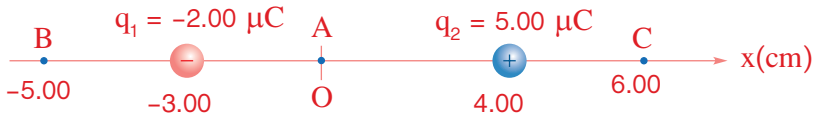
$$= 8.99 \times 10^9 \left(-\frac{|-8.00 \times 10^{-6}|}{0.360^2} + \frac{|5.00 \times 10^{-6}|}{0.160^2}\right)\vec{i}$$

$$\vec{E}_A = 1.20 \times 10^6 \vec{i} \text{ N/C}$$

In conclusion, the resultant electric field at x = 0.360 m has a magnitude of 1.20×10^6 N/C directed along the positive x-axis. Ans

Problem 17.38

Charges $q_1 = -2.00\ \mu C$ and $q_2 = 5.00\ \mu C$ are arranged on the x-axis at $x_1 = -3.00$ cm and $x_2 = 4.00$ cm, respectively, as in Fig. (a). What are the electric fields at points A, B, and C?



(a)

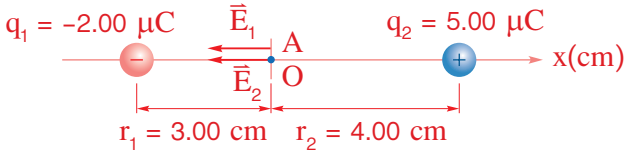
Solution

Each of the electric fields at points A, B, and C is the vector sum of the electric fields from charges q_1 and q_2 at that point. The electric field due to charge q_1 is directed towards the charge since charge q_1 is negative; the electric field due to charge q_2 is directed outward from the charge because of its positive charge. The magnitude of each electric field is

$$E = k_0 \frac{|q|}{r^2}$$

electric field at point A:

In Fig. (b), electric field $\vec{E}_1$ is directed towards charge q_1 along the negative x-axis; electric field $\vec{E}_2$ is directed outward from charge q_2 along the negative x-axis as well. Therefore, the resultant electric field at point A, the sum of electric fields $\vec{E}_1$ and $\vec{E}_2$, is also directed along the negative x-axis and of a magnitude equal to the sum of the magnitudes of electric fields $\vec{E}_1$ and $\vec{E}_2$, as follows:



(b)

$$\vec{E}_A = (E_1 + E_2)(-\vec{i})$$

$$= \left(k_0 \frac{|q_1|}{r_1^2} + k_0 \frac{|q_2|}{r_2^2}\right)(-\vec{i})$$

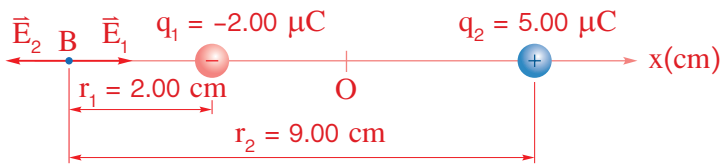
$$= -k_0 \left(\frac{|q_1|}{r_1^2} + \frac{|q_2|}{r_2^2}\right)\vec{i}$$

$$= -8.99 \times 10^9 \left(\frac{|-2.00 \times 10^{-6}|}{(3.00 \times 10^{-2})^2} + \frac{|5.00 \times 10^{-6}|}{(4.00 \times 10^{-2})^2}\right)\vec{i}$$

$$\vec{E}_A = -4.81 \times 10^7 \vec{i} \text{ N/C}$$ Ans

electric field at point B:

In the same way as point A, electric field $\vec{E}_1$ is directed along the positive x-axis; electric field $\vec{E}_2$ is directed along the negative x-axis, as shown in Fig. (c). Electric fields $\vec{E}_1, \vec{E}_2$, and resultant electric field $\vec{E}_B$ are



(c)

$$\vec{E}_1 = k_0 \frac{|q_1|}{r_1^2}\vec{i}$$

$$\vec{E}_2 = k_0 \frac{|q_2|}{r_2^2}(-\vec{i})$$

$$\vec{E}_B = \vec{E}_1 + \vec{E}_2$$

$$= \left(k_0 \frac{|q_1|}{r_1^2}\vec{i}\right) + \left(-k_0 \frac{|q_2|}{r_2^2}\vec{i}\right)$$

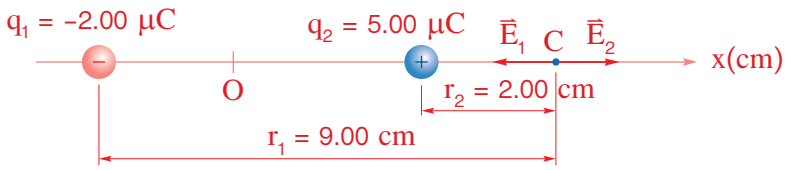
$$= k_0 \left(\frac{|q_1|}{r_1^2} - \frac{|q_2|}{r_2^2}\right)\vec{i}$$

$$= 8.99 \times 10^9 \left(\frac{|-2.00 \times 10^{-6}|}{(2.00 \times 10^{-2})^2} - \frac{|5.00 \times 10^{-6}|}{(9.00 \times 10^{-2})^2} \right) \vec{i}$$

$$\vec{E}_B = 3.94 \times 10^7 \vec{i} \ \text{N/C}$$ Ans

electric field at point C:

Figure (d) shows electric fields $\vec{E}_1$ and $\vec{E}_2$ at point C due to charges q_1 and q_2. Similarly, the resultant electric field $\vec{E}_C$ at point C is given by



(d)

$$\vec{E}_1 = k_0 \frac{|q_1|}{r_1^2}(-\vec{i})$$

$$\vec{E}_2 = k_0 \frac{|q_2|}{r_2^2}\vec{i}$$

$$\vec{E}_C = \vec{E}_1 + \vec{E}_2$$

$$= \left(-k_0 \frac{|q_1|}{r_1^2}\vec{i} \right) + \left(k_0 \frac{|q_2|}{r_2^2}\vec{i} \right)$$

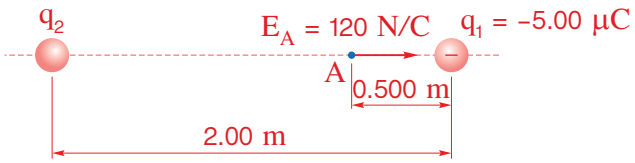
$$= k_0 \left(\frac{-|q_1|}{r_1^2} + \frac{|q_2|}{r_2^2} \right) \vec{i}$$

$$= 8.99 \times 10^9 \left(\frac{-|-2.00 \times 10^{-6}|}{(9.00 \times 10^{-2})^2} + \frac{|5.00 \times 10^{-6}|}{(2.00 \times 10^{-2})^2} \right) \vec{i}$$

$$\vec{E}_C = 1.10 \times 10^8 \vec{i} \ \text{N/C}$$ Ans

Problem 17.39

Charges $q_1 = -5.00\ \mu C$ and q_2 are 2.00 m apart, as shown in Fig. (a). If the electric field at point A, a distance 0.500 m away from charge q_1, has a magnitude of 120 N/C directed to the right, find the magnitude and type of charge q_2.

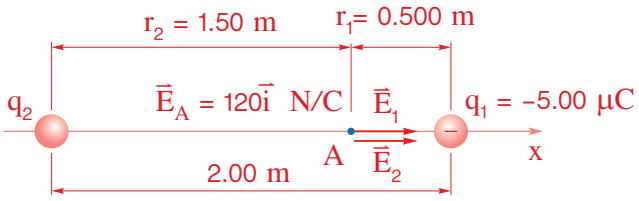


(a)

Solution

Let charges q_1, q_2, and point A be on the x-axis, as in Fig. (b). At point A, the resultant electric field $\vec{E}_A$ is the vector sum of electric fields $\vec{E}_1$ and $\vec{E}_2$ due to charges q_1 and q_2, respectively. Since the resultant electric field at point A has a magnitude of 120 N/C directed to the right,

$$\vec{E}_A = 120\vec{i}\ \text{N/C}$$



(b)

Electric field $\vec{E}_1$ due to charge q_1 points to the right along the positive x-axis towards charge q_1 because q_1 is a negative charge. Therefore,

$$\vec{E}_1 = k_0 \frac{|q_1|}{r_1^2}\vec{i}$$

We now know electric fields $\vec{E}_A$ and $\vec{E}_1$. So, electric field $\vec{E}_2$ at point A can be obtained, which leads to the desired magnitude and type of charge q_2, as follows:

$$\vec{E}_A = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_2 = \vec{E}_A - \vec{E}_1$$

$$= 120\,\vec{i} - k_0 \frac{|q_1|}{r_1^2}\,\vec{i}$$

$$= \left(120 - \frac{8.99 \times 10^9 \times \left|-5.00 \times 10^{-6}\right|}{0.500^2}\right)\vec{i}$$

$$\vec{E}_2 = -1.79(68) \times 10^5\,\vec{i}\ \text{N/C}$$

The result shows that electric field $\vec{E}_2$ is directed along the negative x-axis, indicated by unit vector $-\vec{i}$. It means that the electric field points toward charge q_2, therefore, q_2 is a negative charge.

The magnitude 1.79(68) $\times$ 10^5 N/C of electric field $\vec{E}_2$ can be used to find the magnitude of charge q_2:

$$E_2 = k_0 \frac{|q_2|}{r_2^2}$$

$$|q_2| = \frac{E_2 r_2^2}{k_0}$$

$$= \frac{1.79(68) \times 10^5 \times 1.50^2}{8.99 \times 10^9}$$

$$= 4.50 \times 10^{-5}\ \text{C}$$

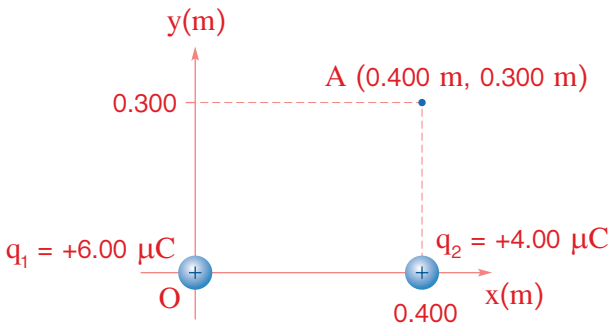
$$= 45.0 \times 10^{-6}\ \text{C}$$

$$|q_2| = 45.0\ \mu\text{C}$$

In conclusion, charge q_2 is $-45.0\ \mu$C. Ans

Problem 17.40

Charges q_1 = +6.00 μC and q_2 = +4.00 μC are located on the x-axis, as in Fig. (a). Find the resultant electric field at point A at position (0.400 m, 0.300 m).

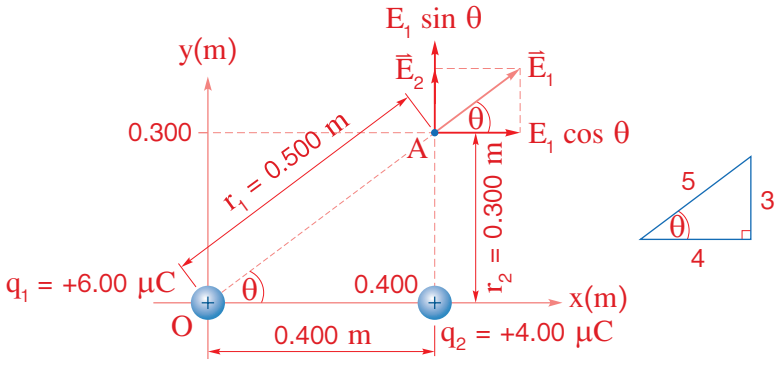


(a)

Solution

At point A, the resultant electric field is the vector sum of electric fields $\vec{E}_1$ and $\vec{E}_2$ due to charges q_1 and q_2, as in Fig. (b). First, find electric fields $\vec{E}_1$ and $\vec{E}_2$, then add them to get the resultant electric field $\vec{E}_A$:

$$\vec{E}_A = \vec{E}_1 + \vec{E}_2 \qquad \ldots (1)$$



(b)

In Fig. (b), electric field $\vec{E}_1$ points outward from charge q_1, making an angle θ with the positive x-axis. The right triangle in the figure already provides the data for the angle θ. Electric field $\vec{E}_1$ is written in its x- and y-components as

$$\vec{E}_1 = E_1 \cos\theta\, \vec{i} + E_1 \sin\theta\, \vec{j} \qquad \text{... (2)}$$

where $$E_1 = k_0 \frac{|q_1|}{r_1^2}$$

$$r_1 = \sqrt{0.400^2 + 0.300^2}$$

$$= 0.500 \text{ m}$$

Electric field $\vec{E}_2$ points outward from charge q_2 in the positive y-axis,

$$\vec{E}_2 = E_2 \vec{j} \qquad \text{... (3)}$$

where $$E_2 = k_0 \frac{|q_2|}{r_2^2}$$

$$r_2 = 0.300 \text{ m}$$

Substitute $\vec{E}_1$ and $\vec{E}_2$ from Eqs. (2) and (3) into Eq. (1) to find the resultant electric field at point A:

$$\vec{E}_A = \vec{E}_1 + \vec{E}_2$$

$$= (E_1 \cos\theta\, \vec{i} + E_1 \sin\theta\, \vec{j}) + E_2 \vec{j}$$

$$= E_1 \cos\theta\, \vec{i} + (E_1 \sin\theta + E_2)\, \vec{j}$$

$$= k_0 \frac{|q_1|}{r_1^2} \cos\theta\, \vec{i} + \left(k_0 \frac{|q_1|}{r_1^2} \sin\theta + k_0 \frac{|q_2|}{r_2^2} \right) \vec{j}$$

$$= 8.99 \times 10^9 \left(\frac{|+6.00 \times 10^{-6}|}{0.500^2} \times \frac{4}{5} \right) \vec{i} +$$

$$8.99 \times 10^9 \left(\frac{|+6.00 \times 10^{-6}|}{0.500^2} \times \frac{3}{5} + \frac{|+4.00 \times 10^{-6}|}{0.300^2} \right) \vec{j}$$

$$\vec{E}_A = 1.73 \times 10^5\, \vec{i} + 5.29 \times 10^5\, \vec{j} \text{ N/C} \qquad \text{Ans}$$

The magnitude and direction of the resultant electric field $\vec{E}_A$ are

$$E_A = \sqrt{(1.73 \times 10^5)^2 + (5.29 \times 10^5)^2}$$

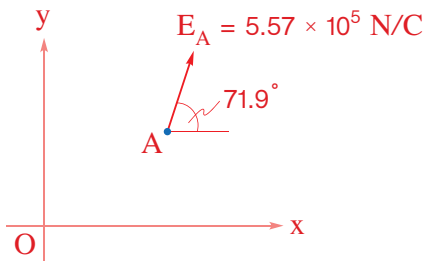
$$E_A = 5.57 \times 10^5 \text{ N/C}$$

$$\tan \alpha = \frac{5.29 \times 10^5}{1.73 \times 10^5}$$

$$\alpha = \tan^{-1}\left(\frac{5.29 \times 10^5}{1.73 \times 10^5}\right)$$

$$\alpha = 71.9^\circ$$

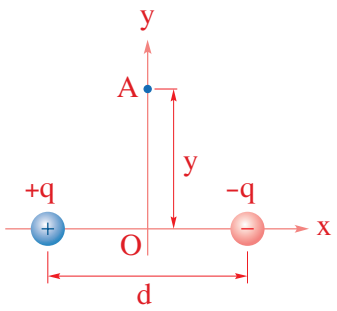
In conclusion, the resultant electric field at point A has a magnitude of 5.57×10^5 N/C directed a 71.9° angle above the horizontal, as in Fig. (c). Ans



(c)

Problem 17.41

Two point charges have the same magnitude q and opposite signs, placed a distance d apart, as shown in Fig. (a). What is the resultant electric field at point A on the perpendicular bisector of the line connecting the two charges, a distance y from the origin O?

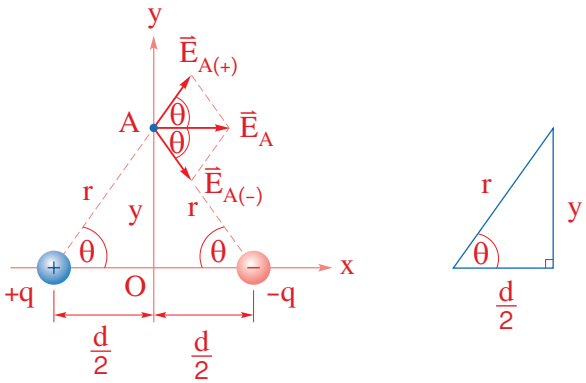


(a)

Solution

In Fig. (b), at point A, electric field $\vec{E}_{A(+)}$ due to charge +q is directed outward from charge +q, making an angle θ above the positive x-axis; electric field $\vec{E}_{A(-)}$ due to charge −q points toward the charge, making the same angle θ but below the positive x-axis. The magnitudes of both fields are equal and given by

$$E_{A(+)} = E_{A(-)} = k_0 \frac{q}{r^2} \qquad \ldots (1)$$



(b)

To find distance r, we use the Pythagorean Theorem on the right triangle in Fig. (b), as follows:

$$r^2 = \left(\frac{d}{2}\right)^2 + y^2 \qquad \ldots (2)$$

Substituting r^2 from Eq. (2) into Eq. (1), we get

$$E_{A(+)} = E_{A(-)} = k_0 \frac{q}{\left(\frac{d}{2}\right)^2 + y^2} \quad \ldots (3)$$

Since electric fields $\vec{E}_{A(+)}$ and $\vec{E}_{A(-)}$ have equal magnitude and make the same angle θ above and below the positive x-axis, the y-components of electric fields $\vec{E}_{A(+)}$ and $\vec{E}_{A(-)}$ cancel to zero, and the resultant electric field $\vec{E}_A$ is equal to the sum of the x-components.

Because the x-components of electric fields $\vec{E}_{A(+)}$ and $\vec{E}_{A(-)}$ are in the same positive x-direction, the resultant electric field $\vec{E}_A$ is also in that direction and has a magnitude of

$$\begin{aligned} E_A &= E_{A(+)} \cos\theta + E_{A(-)} \cos\theta \\ &= 2E_{A(+)} \cos\theta, \quad (E_{A(+)} = E_{A(-)}) \end{aligned} \quad \ldots (4)$$

From the right triangle in Fig. (b),

$$\cos\theta = \frac{\frac{d}{2}}{r} = \frac{d}{2\left(\left(\frac{d}{2}\right)^2 + y^2\right)^{\frac{1}{2}}} \quad \ldots (5)$$

Substitute $E_{A(+)}$ from Eq. (3) and $\cos\theta$ from Eq. (5) into Eq. (4):

$$E_A = 2\left(k_0 \frac{q}{\left(\frac{d}{2}\right)^2 + y^2}\right)\left(\frac{d}{2\left(\left(\frac{d}{2}\right)^2 + y^2\right)^{\frac{1}{2}}}\right)$$

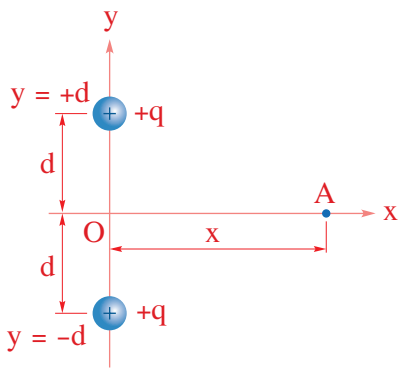
$$E_A = k_0 \frac{qd}{\left(\left(\frac{d}{2}\right)^2 + y^2\right)^{\frac{3}{2}}}$$

In conclusion, the resultant electric field at point A has a magnitude of $k_0 \dfrac{qd}{\left(\left(\frac{d}{2}\right)^2 + y^2\right)^{\frac{3}{2}}}$ in the positive x-direction or written in vector form as

$$\vec{E}_A = k_0 \frac{qd}{\left(\left(\frac{d}{2}\right)^2 + y^2\right)^{\frac{3}{2}}} \vec{i}$$ Ans

Problem 17.42

Two point charges +q are fixed on the y-axis at y = ±d, as shown in Fig. (a). Find the resultant electric field at point A on the x-axis, a distance x from the origin O, and the position of point A which maximizes the electric field.



(a)

Solution

Since the resultant electric field at point A is the vector sum of electric fields due to the two charges +q, we begin by finding the electric field from each charge and then add them vectorially to get the resultant electric field. Next, do the derivative of the resultant electric field with respect to x, then set to zero and solve for the required coordinate x that maximizes the electric field, as follows:

In Fig. (b), given the electric fields at point A due to the two charges +q represented by electric fields $\vec{E}_1$ and $\vec{E}_2$ outward from the charges. Since each charge

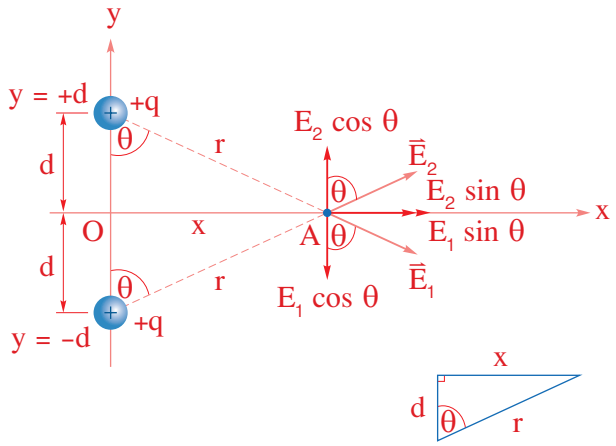
is at the same distance d from the origin O, point A is equidistant from each charge at distance r, and electric fields $\vec{E}_1$ and $\vec{E}_2$ at point A have the same magnitude of

$$E_1 = E_2 = k_0 \frac{q}{r^2} \quad \ldots (1)$$

where distance r is obtained by using the Pythagorean Theorem on the right triangle in Fig. (b):

$$r^2 = d^2 + x^2$$

$$r = \left(d^2 + x^2\right)^{\frac{1}{2}} \quad \ldots (2)$$



(b)

Electric fields $\vec{E}_1$ and $\vec{E}_2$ at point A both make an angle θ with the vertical, as shown in Fig. (b). When resolving these two fields into their horizontal and vertical components, it is found that the sum of the vertical components is zero (two vectors of magnitudes $E_1 \cos\theta$ and $E_2 \cos\theta$ in the opposite direction and $E_1 = E_2$). Therefore, the resultant electric field at point A is just the sum of the horizontal components.

From Fig. (b), the horizontal components of the electric fields are of magnitudes $E_1 \sin\theta$ and $E_2 \sin\theta$ directed in the same positive x-direction, so the resultant electric field is also in that direction and has a magnitude of

$$E = E_1 \sin\theta + E_2 \sin\theta$$

$$= E_1 \sin\theta + E_1 \sin\theta \ , \quad E_1 = E_2$$

$$E = 2E_1 \sin\theta \quad \ldots (3)$$

From angle θ on the right triangle in Fig. (b), $\sin\theta = \frac{x}{r}$. Substitute this into Eq. (3), and use the value of E_1 from Eq. (1), we get

$$E = 2\left(k_0 \frac{q}{r^2}\right)\left(\frac{x}{r}\right)$$

$$E = 2k_0 \frac{qx}{r^3} \qquad \ldots (4)$$

Substitute r from Eq. (2) into Eq. (4):

$$E = 2k_0 \frac{qx}{(x^2 + d^2)^{\frac{3}{2}}}$$

In conclusion, the resultant electric field at point A has a magnitude of $2k_0 \frac{qx}{(x^2 + d^2)^{\frac{3}{2}}}$ in the positive x-direction or written in vector form as

$$\vec{E} = 2k_0 \frac{qx}{(x^2 + d^2)^{\frac{3}{2}}} \vec{i} \qquad \text{Ans}$$

The resultant electric field is maximum if $\frac{dE}{dx} = 0$, so

$$\frac{dE}{dx} = \frac{d}{dx}\left(\frac{2k_0 qx}{(x^2 + d^2)^{\frac{3}{2}}}\right)$$

$$= \frac{2k_0 q(x^2 + d^2)^{\frac{3}{2}} - 2k_0 qx\left(\frac{3}{2}\right)(x^2 + d^2)^{\frac{1}{2}}(2x)}{(x^2 + d^2)^{\frac{3}{2}\times 2}}$$

$$= \frac{2k_0 q}{(x^2 + d^2)^{\frac{3}{2}}} - \frac{6k_0 qx^2}{(x^2 + d^2)^{\frac{5}{2}}}$$

$$= 0$$

Solve the above expression for x:

$$\frac{2k_0q}{(x^2 + d^2)^{\frac{3}{2}}} = \frac{6k_0qx^2}{(x^2 + d^2)^{\frac{5}{2}}}$$

$$x^2 + d^2 = 3x^2$$

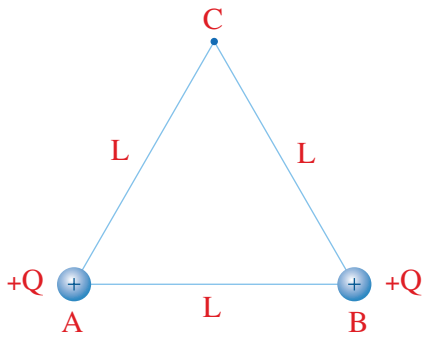
$$2x^2 = d^2$$

$$x = \pm\frac{d}{\sqrt{2}}$$

In conclusion, the resultant electric field is maximum at $x = \pm\frac{d}{\sqrt{2}}$. Ans

Problem 17.43

Two point charges +Q are placed at points A and B on an equilateral triangle ABC of side L, as in Fig. (a). What is the electric field at point C?

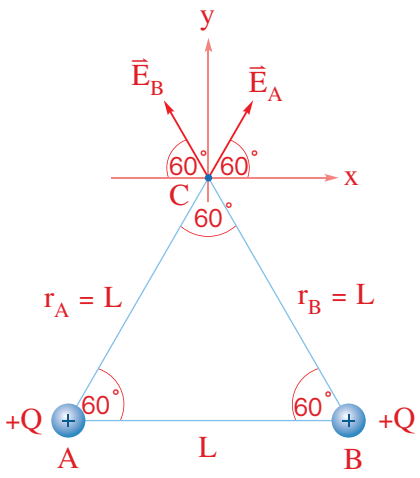


(a)

Solution

The resultant electric field at point C is the vector sum of electric fields $\vec{E}_A$ and $\vec{E}_B$ due to charges +Q at points A and B, respectively, as shown in Fig. (b). Since the charges at points A and B are equal and these two points are equidistant from point C by a distance L, the magnitude of electric fields $\vec{E}_A$ and $\vec{E}_B$ at point C are the same:

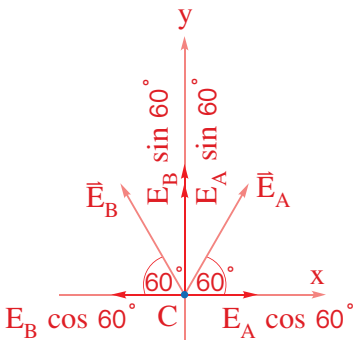
$$E_A = E_B = k_0\frac{|+Q|}{L^2} = \frac{k_0Q}{L^2} \quad \ldots (1)$$



(b)

Because each of the internal angles of an equilateral triangle is 60°, electric field $\vec{E}_A$ is directed away from charge +Q at point A at an angle of 60° above the positive x-axis. Similarly, electric field $\vec{E}_B$ is directed outward charge +Q at point B at a 60° angle above the negative x-axis, as shown in Fig. (b).

From the patterns of electric fields $\vec{E}_A$ and $\vec{E}_B$ in Fig. (b), we sum them by resolving into their components along the x- and y-axes, as in Fig. (c).



(c)

From Fig. (c), we see that, on the x-axis, there are two electric fields of magnitudes $E_A \cos 60°$ and $E_B \cos 60°$ in the opposite direction. Since $E_A = E_B$, these two fields cancel to zero, and the resultant x-component is zero. Therefore, the resultant electric field at point C is just the sum of the y-components.

Consider along the y-axis, the resultant electric field at point C is the sum of the electric fields of magnitudes $E_A \sin 60°$ and $E_B \sin 60°$ in the same positive

y-direction, therefore

$$\vec{E}_C = (E_A \sin 60^\circ + E_B \sin 60^\circ)\,\vec{j}$$

$$= (E_A \sin 60^\circ + E_A \sin 60^\circ)\,\vec{j}, \quad E_A = E_B$$

$$\vec{E}_C = 2E_A \sin 60^\circ\,\vec{j} \qquad \ldots (2)$$

Substitute E_A from Eq. (1) into Eq. (2):

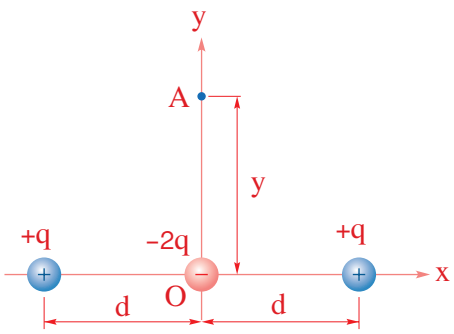
$$\vec{E}_C = 2\left(\frac{k_0 Q}{L^2}\right)\frac{\sqrt{3}}{2}\,\vec{j}$$

$$\vec{E}_C = \frac{\sqrt{3}k_0 Q}{L^2}\,\vec{j}$$

From the above result, it can be concluded that the electric field at point C has a magnitude of $\frac{\sqrt{3}k_0 Q}{L^2}$ in the upward direction. Ans

Problem 17.44

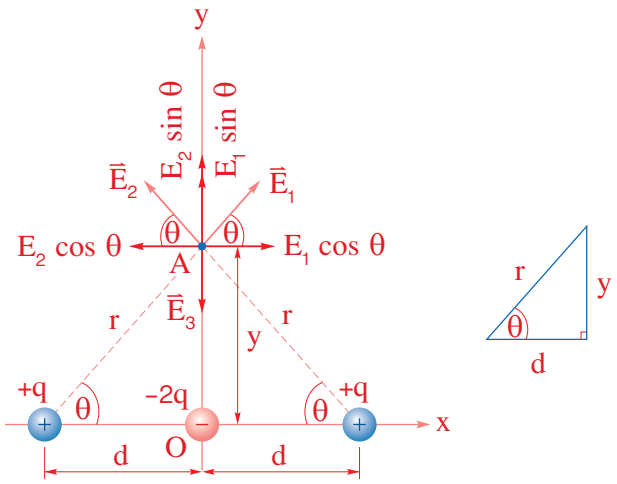
Three point charges are placed on the x-axis, as shown in Fig. (a). From the figure, charge -2q is at the origin O, and the other two charges of +q are away from the origin by the same distance d. Find the resultant electric field at point A, which is on the y-axis, a distance y from the origin O.



(a)

Solution

Figure (b) shows the electric fields at point A, which consist of electric fields $\vec{E}_1$ and $\vec{E}_2$ due to the two charges +q and electric field $\vec{E}_3$ due to charge –2q.



(b)

Since point A is equidistant from each charge +q, electric fields $\vec{E}_1$ and $\vec{E}_2$ have the same magnitude of

$$E_1 = E_2 = \frac{k_0 q}{r^2} \qquad \ldots (1)$$

where $r^2 = d^2 + y^2$

Because the distance between point A and charge –2q is y, the magnitude of electric field $\vec{E}_3$ is given by

$$E_3 = k_0 \frac{|-2q|}{y^2} = \frac{2k_0 q}{y^2} \qquad \ldots (2)$$

Electric field $\vec{E}_3$ is directed towards charge –2q along the negative y-axis; electric fields $\vec{E}_1$ and $\vec{E}_2$ are directed outward from each charge +q at an angle θ with the horizontal, as shown in Fig. (b).

The resultant electric field at point A is the vector sum of electric fields $\vec{E}_1, \vec{E}_2$ and $\vec{E}_3$. To find the sum, we begin by resolving electric fields $\vec{E}_1$ and $\vec{E}_2$ into their horizontal and vertical components, as in Fig. (b). Because of $E_1 = E_2$, the resultant

horizontal component, which consists of two fields of magnitudes $E_1 \cos\theta$ and $E_2 \cos\theta$ in the opposite direction, is equal to zero. The resultant electric field at point A is just the sum of the vertical components, as follows:

$$\vec{E}_A = (E_1 \sin\theta + E_2 \sin\theta - E_3)\,\vec{j}$$

$$\vec{E}_A = (2E_1 \sin\theta - E_3)\,\vec{j}\,, \quad E_1 = E_2 \qquad \ldots (3)$$

Substitute E_1 and E_3 from Eqs. (1) and (2) into Eq. (3), then use $\sin\theta = \frac{y}{r}$ from the right triangle in Fig. (b):

$$\vec{E}_A = \left(2 \times \frac{k_0 q}{r^2} \times \frac{y}{r} - \frac{2k_0 q}{y^2}\right)\vec{j}$$

$$= \left(2k_0 q \frac{y}{r^3} - 2k_0 q \frac{1}{y^2}\right)\vec{j}$$

$$= \left(2k_0 q \frac{y}{(d^2 + y^2)^{\frac{3}{2}}} - 2k_0 q \frac{1}{y^2}\right)\vec{j}$$

$$= \left(2k_0 q \frac{y}{y^3\left(1 + \frac{d^2}{y^2}\right)^{\frac{3}{2}}} - 2k_0 q \frac{1}{y^2}\right)\vec{j}$$

$$\vec{E}_A = \frac{2k_0 q}{y^2}\left(\frac{1}{\left(1 + \frac{d^2}{y^2}\right)^{\frac{3}{2}}} - 1\right)\vec{j}$$

Because $1 + \frac{d^2}{y^2} > 1$, $1 / \left(1 + \frac{d^2}{y^2}\right)^{\frac{3}{2}} < 1$. By this, the above expression for the resultant electric field $\vec{E}_A$ can be rewritten to express the magnitude and direction

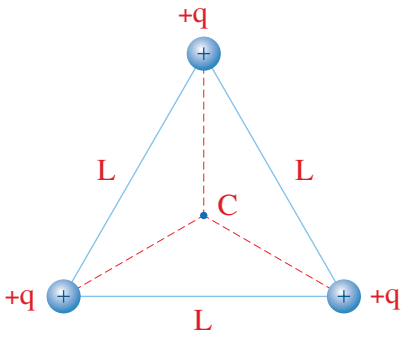
clearly, as the following:

$$\vec{E}_A = -\frac{2k_0q}{y^2}\left(1 - \frac{1}{\left(1 + \frac{d^2}{y^2}\right)^{\frac{3}{2}}}\right)\vec{j}$$

In conclusion, the resultant electric field at point A is directed towards charge −2q along the negative y-axis and has a magnitude of $\frac{2k_0q}{y^2}\left(1 - \frac{1}{\left(1 + \frac{d^2}{y^2}\right)^{\frac{3}{2}}}\right)$. Ans

Problem 17.45

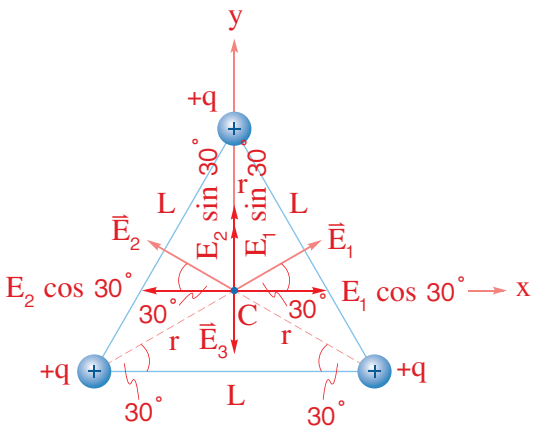
Three point charges +q are placed at the corners of an equilateral triangle of side L, as shown in Fig. (a). What is the resultant electric field at point C, the center of the triangle?



(a)

Solution

The resultant electric field at point C is the vector sum of electric fields $\vec{E}_1, \vec{E}_2$ and $\vec{E}_3$ at that point due to the three charges +q, as shown in Fig. (b).



(b)

The center of the triangle is at the same distance r from each charge +q, as in Fig. (b). Therefore, electric fields $\vec{E}_1, \vec{E}_2$ and $\vec{E}_3$ have the same magnitude of

$$E_1 = E_2 = E_3 = \frac{k_0 q}{r^2}$$

Since charge +q is positive, electric field $\vec{E}_1$ is directed outward from the charge at the bottom left corner with a 30° angle above the positive x-axis, electric field $\vec{E}_2$ is directed outward from the charge at the bottom right corner with a 30° angle above the negative x-axis, and electric field $\vec{E}_3$ is directed outward from the charge at the top corner in the downward direction, as shown in Fig. (b).

At point C, the resultant electric field can be obtained by resolving electric fields $\vec{E}_1$ and $\vec{E}_2$ into their horizontal and vertical components. From Fig. (b), on the horizontal, there are two electric field components of magnitudes $E_1 \sin 30^\circ$ and $E_2 \sin 30^\circ$ in the opposite direction. Since $E_1 = E_2$, the sum of all the horizontal components is zero.

The vertical consists of two electric field components of magnitudes $E_1 \sin 30^\circ$ and $E_2 \sin 30^\circ$ in the same upward direction, and the electric field of magnitude E_3 in the downward direction. Since the resultant horizontal component is zero, the resultant electric field at point C is the vector sum of the three fields on the vertical as mentioned:

$$\vec{E}_C = (E_1 \sin 30^\circ + E_2 \sin 30^\circ - E_3)\,\vec{j}$$

$$= (2E_1 \sin 30^\circ - E_3)\,\vec{j}, \quad E_1 = E_2$$

$$= (E_1 - E_3)\,\vec{j}, \quad \sin 30^\circ = \frac{1}{2}$$

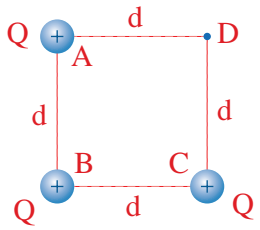
$$= (E_1 - E_1)\,\vec{j}, \quad E_1 = E_3$$

$$\vec{E}_C = 0$$

In conclusion, the resultant electric field at the center of the triangle is equal to zero. Ans

Problem 17.46

Three point charges, each $Q = 3.00 \times 10^{-9}$ C, are fixed at the corners of a square with side d = 1.20 m, as shown in Fig. (a). Find the electric field at corner D.



(a)

Solution

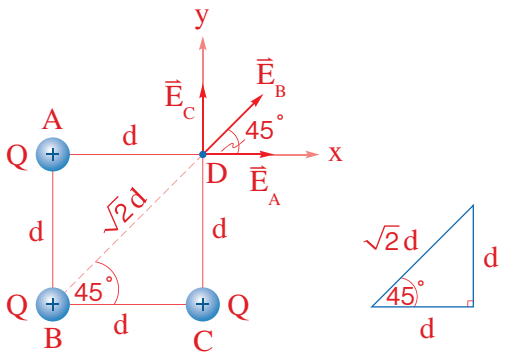
Electric field $\vec{E}_D$ at point D is the vector sum of electric fields $\vec{E}_A, \vec{E}_B$ and $\vec{E}_C$ due to charges Q at points A, B, and C, respectively:

$$\vec{E}_D = \vec{E}_A + \vec{E}_B + \vec{E}_C \qquad \ldots (1)$$

Find electric fields $\vec{E}_A, \vec{E}_B$ and $\vec{E}_C$ from the data in Fig. (b), then substitute them into Eq. (1) to get the desired electric field $\vec{E}_D$, as follows:

From Fig. (b), electric field $\vec{E}_A$ due to charge Q at point A is directed away from point A along the positive x-axis, and points A and D are separated by a distance d, therefore

$$\vec{E}_A = k_0 \frac{Q}{d^2} \vec{i} \quad \text{... (2)}$$



(b)

Electric field $\vec{E}_C$ is directed upwards along the positive y-axis outward from charge Q at point C, and points C and D are separated by the same distance d, just as point A, so

$$\vec{E}_C = k_0 \frac{Q}{d^2} \vec{j} \quad \text{... (3)}$$

Electric field $\vec{E}_B$ is directed along the diagonal outward from the charge at point B, making a 45° angle with the positive x-axis. Since points B and D are separated by a distance $\sqrt{2}\,d$ (obtained by applying the Pythagorean Theorem on the right triangle in Fig. (b)), electric field $\vec{E}_B$ can be written in the x- and y-components as

$$\vec{E}_B = E_B \cos 45^\circ \vec{i} + E_B \sin 45^\circ \vec{j}$$

$$= k_0 \frac{Q}{(\sqrt{2}d)^2} \cos 45^\circ \vec{i} + k_0 \frac{Q}{(\sqrt{2}d)^2} \sin 45^\circ \vec{j}$$

$$\vec{E}_B = k_0 \frac{Q}{2d^2} \cos 45^\circ \vec{i} + k_0 \frac{Q}{2d^2} \sin 45^\circ \vec{j} \quad \text{... (4)}$$

Substitute $\vec{E}_A$, $\vec{E}_B$ and $\vec{E}_C$ from Eqs. (2) through (4) into Eq. (1) to find electric field $\vec{E}_D$ at point D:

$$\vec{E}_D = \vec{E}_A + \vec{E}_B + \vec{E}_C$$

$$= \left(k_0 \frac{Q}{d^2}\vec{i}\right) + \left(k_0 \frac{Q}{2d^2}\cos 45^\circ \vec{i} + k_0 \frac{Q}{2d^2}\sin 45^\circ \vec{j}\right) + k_0 \frac{Q}{d^2}\vec{j}$$

$$= \left(k_0 \frac{Q}{d^2} + k_0 \frac{Q}{2d^2}\cos 45^\circ\right)\vec{i} + \left(k_0 \frac{Q}{d^2} + k_0 \frac{Q}{2d^2}\sin 45^\circ\right)\vec{j}$$

$$= k_0 \frac{Q}{d^2}\left(1 + \frac{1}{2} \times \frac{\sqrt{2}}{2}\right)\vec{i} + k_0 \frac{Q}{d^2}\left(1 + \frac{1}{2} \times \frac{\sqrt{2}}{2}\right)\vec{j}$$

$$\vec{E}_D = k_0 \frac{Q}{d^2}\left(1 + \frac{\sqrt{2}}{4}\right)\vec{i} + k_0 \frac{Q}{d^2}\left(1 + \frac{\sqrt{2}}{4}\right)\vec{j} \qquad \text{... (5)}$$

Find the value of $k_0 \frac{Q}{d^2}\left(1 + \frac{\sqrt{2}}{4}\right)$ and substitute it into Eq. (5):

$$k_0 \frac{Q}{d^2}\left(1 + \frac{\sqrt{2}}{4}\right) = 8.99 \times 10^9 \times \frac{3.00 \times 10^{-9}}{1.20^2}\left(1 + \frac{\sqrt{2}}{4}\right)$$

$$= 25.4 \text{ N/C}$$

Therefore,

$$\vec{E}_D = 25.4\,\vec{i} + 25.4\,\vec{j} \text{ N/C} \qquad \text{Ans}$$

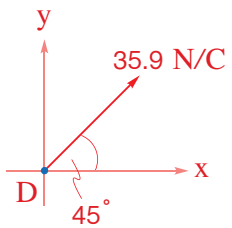
The magnitude and direction of electric field $\vec{E}_D$ is given by

$$E_D = \sqrt{25.4^2 + 25.4^2}$$

$$E_D = 35.9 \text{ N/C}$$

$$\tan\theta = \frac{25.4}{25.4} = 1$$

$$\theta = 45^\circ$$

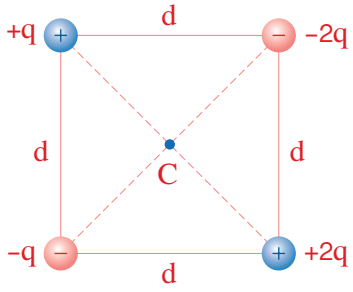


(c)

It can be concluded that the electric field at point D has a magnitude of 35.9 N/C directed at a 45° angle above the positive x-axis, as shown in Fig. (c). Ans

Problem 17.47

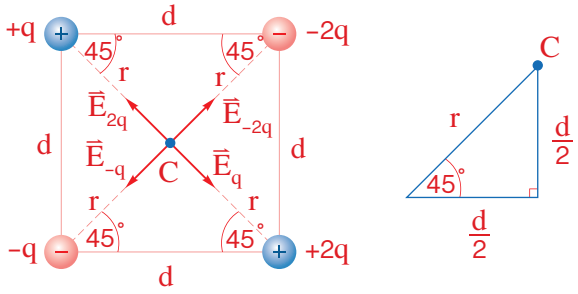
Four point charges are placed at the corners of a square of side d, as shown in Fig. (a). What is the resultant electric field at center C of the square?



(a)

Solution

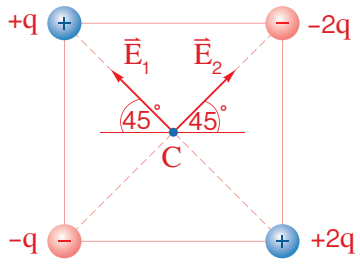
The resultant electric field at center C is the vector sum of the electric fields at that point due to all the four charges, as shown in Fig. (b).



(b)

From Fig. (b), electric fields $\vec{E}_q$ and $\vec{E}_{2q}$ are directed outward from the charges +q and +2q along the diagonal connecting the charges. It makes these two electric fields opposite in direction. So, we can add them up to electric field $\vec{E}_1$, as shown in Fig. (c), and the magnitude E_1 is given by

$$E_1 = E_{2q} - E_q \qquad \ldots (1)$$



(c)

Since center C and each charge are separated by distance r, therefore

$$E_q = k_0 \frac{q}{r^2}$$

$$E_{2q} = k_0 \frac{2q}{r^2} = 2k_0 \frac{q}{r^2}$$

Substituting E_q and E_{2q} into Eq. (1) gives

$$E_1 = 2k_0 \frac{q}{r^2} - k_0 \frac{q}{r^2} = k_0 \frac{q}{r^2}$$

In the same way, electric fields $\vec{E}_{-q}$ and $\vec{E}_{-2q}$ are directed towards the charges −q and −2q along the diagonal joining the charges, as shown in Fig. (b). Since these two fields are in the opposite direction, they add up to electric field $\vec{E}_2$, as in Fig. (c). Magnitude E_2 can be found like magnitude E_1, as follows:

$$E_2 = E_{-2q} - E_{-q}$$

$$= k_0 \frac{|-2q|}{r^2} - k_0 \frac{|-q|}{r^2}$$

$$E_2 = k_0 \frac{q}{r^2}$$

The resultant electric field at center C now is the vector sum of electric fields $\vec{E}_1$ and $\vec{E}_2$, as in Fig. (c). These two electric fields have the same magnitude. Therefore, when resolving them into their horizontal and vertical components, as in Fig. (d), we see that the vector sum of all the horizontal components is zero ($E_1 \cos 45^\circ = E_2 \cos 45^\circ$ and of opposite direction). The resultant electric field is just the vector sum of all the vertical components, which are directed upward, so the magnitude of

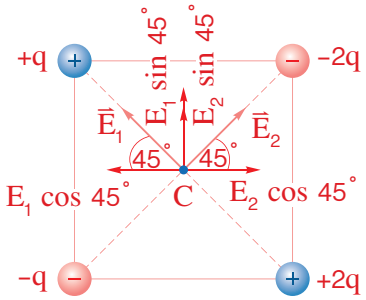
the resultant electric field at point C is

$$E_C = E_1 \sin 45^\circ + E_2 \sin 45^\circ$$

$$= 2E_1 \sin 45^\circ, \quad E_1 = E_2$$

$$= 2\left(k_0 \frac{q}{r^2}\right) \times \frac{\sqrt{2}}{2}, \quad E_1 = k_0 \frac{q}{r^2}$$

$$E_C = \sqrt{2}\, k_0 \frac{q}{r^2} \qquad \text{... (2)}$$



(d)

From the right triangle in Fig. (b),

$$r^2 = \left(\frac{d}{2}\right)^2 + \left(\frac{d}{2}\right)^2 = \frac{d^2}{2}$$

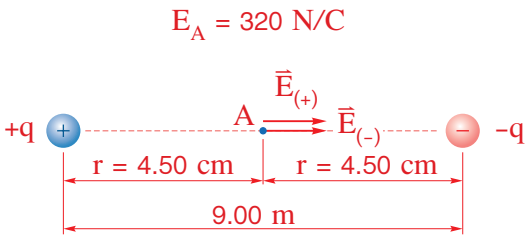
Substitute the value of r^2 into Eq. (2):

$$E_C = \sqrt{2}\, k_0 \frac{q}{\frac{d^2}{2}} = \frac{2\sqrt{2}k_0 q}{d^2}$$

In conclusion, the resultant electric field at the square's center has a magnitude of $\frac{2\sqrt{2}k_0 q}{d^2}$ in the upward direction. Ans

Problem 17.48

An electric field at the midpoint of a pair of charges, which have the same magnitude and opposite signs, is equal to 320 N/C. If the charges are 9.00 cm apart, find the magnitude of each charge.



Given each charge has a magnitude q and they are placed 9.00 cm apart, as shown in the figure. From the figure, point A is the midpoint of the two charges, +q and −q, and has an electric field of magnitude E_A = 320 N/C. At point A, the electric field is the vector sum of electric fields $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ due to charges +q and −q. Since electric fields $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ are in the same direction, magnitude E_A is the algebraic sum of magnitudes $E_{(+)}$ and $E_{(-)}$, as follows:

$$E_A = E_{(+)} + E_{(-)} \qquad \text{... (1)}$$

where

$$E_{(+)} = k_0 \frac{|+q|}{r^2} = k_0 \frac{q}{r^2}$$

$$E_{(-)} = k_0 \frac{|-q|}{r^2} = k_0 \frac{q}{r^2}$$

Substitute $E_{(+)}$ and $E_{(-)}$ into Eq. (1):

$$E_A = k_0 \frac{q}{r^2} + k_0 \frac{q}{r^2}$$

$$= \frac{2k_0 q}{r^2}$$

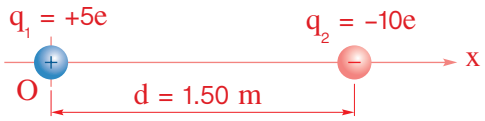
$$q = \frac{r^2 E_A}{2k_0} \qquad \text{...(2)}$$

Substitute E_A = 320 N/C and r = 4.50 cm = 4.50×10^{-2} m into Eq. (2) to calculate magnitude q of the charge, as follows:

$$q = \frac{(4.50 \times 10^{-2})^2 \times 320}{2 \times 8.99 \times 10^9} = 3.60 \times 10^{-11} \text{ C}$$ Ans

Problem 17.49

Charges q_1 = +5e and q_2 = −10e are placed on the x-axis, a distance d = 1.50 m apart, as shown in Fig. (a). Find the position where the resultant electric field due to both charges is equal to zero.

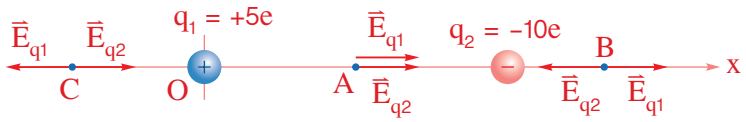


(a)

Solution

Points in a region of charges q_1 and q_2 would have two electric fields due to the two charges. So, the resultant electric field at any point is zero, only if the electric fields from charges q_1 and q_2 have the same magnitude and opposite directions. From this concept, it can be seen that points that are not on the line passing through the charges, that is, not on the x-axis, have no chance to have resultant electric fields equal to zero. It is because the electric fields due to charges q_1 and q_2 cannot be in opposite directions.

The position where the resultant electric field can be zero must be on the line containing the two charges, here the x-axis, as in Fig. (b). From this figure, consider points A, B, and C. Begin at point A between the charges. At this point, electric fields $\vec{E}_{q1}$ and $\vec{E}_{q2}$ are directed outward from charge q_1 and towards charge q_2, respectively; both are directed to the right. Thus, point A or points on the line between the charges cannot have the resultant electric field equal to zero.



(b)

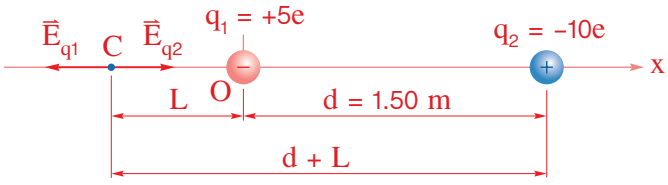
For point B, which is to the right of charge q_2, electric fields $\vec{E}_{q1}$ and $\vec{E}_{q2}$ are in the opposite direction. However, the magnitude of the electric field is proportional to the charges' magnitude and conversely proportional to the square of the distance between the charges $\left(E \propto |q| \text{ and } E \propto \frac{1}{r^2}\right)$. Therefore, at point B closer to charge q_2 with greater magnitude of charge, electric field $\vec{E}_{q2}$ always has a greater magnitude than electric field $\vec{E}_{q1}$. The resultant electric field at point B cannot be zero as well.

The position where the resultant electric field can be zero is at point C to the left of charge q_1. It is because, at this point, electric fields $\vec{E}_{q1}$ and $\vec{E}_{q2}$ are in the opposite direction, and the point is also closer to charge q_1 with less magnitude of charge, the magnitudes of electric fields $\vec{E}_{q1}$ and $\vec{E}_{q2}$ can be equal. Next, we would seek the position of point C that has the resultant electric field equal to zero.

Given point C away from the left of charge q_1 by distance L, as shown in Fig. (c). At this point, electric fields $\vec{E}_{q1}$ and $\vec{E}_{q2}$ must be equal in magnitude and opposite in direction. So, find the magnitudes of these two electric fields, then set them equal to evaluate distance L, as follows:

$$E_{q1} = k_0 \frac{|q_1|}{L^2} = k_0 \frac{q_1}{L^2} \quad , \quad |q_1| = q_1$$

$$E_{q2} = k_0 \frac{|q_2|}{(d + L)^2}$$



(c)

The resultant electric field at point C is equal to zero when

$$E_{q1} = E_{q2}$$

$$k_0 \frac{q_1}{L^2} = k_0 \frac{|q_2|}{(d + L)^2}$$

$$\left(\frac{d + L}{L}\right)^2 = \frac{|q_2|}{q_1}$$

$$\frac{d}{L} + 1 = \frac{\sqrt{|q_2|}}{\sqrt{q_1}}$$

$$\frac{d}{L} = \frac{\sqrt{|q_2|}}{\sqrt{q_1}} - 1$$

$$= \frac{\sqrt{|q_2|} - \sqrt{q_1}}{\sqrt{q_1}}$$

$$L = \frac{\sqrt{q_1}}{\sqrt{|q_2|} - \sqrt{q_1}} d$$

$$= \frac{\sqrt{5e}}{\sqrt{|-10e|} - \sqrt{5e}} \times 1.50$$

$$= \frac{\sqrt{e}}{\sqrt{e}} \times \frac{\sqrt{5}}{\sqrt{10} - \sqrt{5}} \times 1.50$$

$$= \frac{\sqrt{5}}{\sqrt{10} - \sqrt{5}} \times 1.50$$

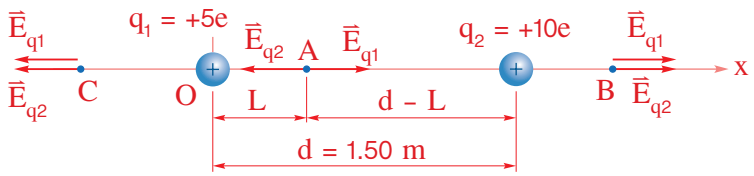
$$L = 3.62 \text{ m}$$

In conclusion, the position where the resultant electric field equals zero is on the left of charge q_1 by a distance of 3.62 m or at coordinate x = –3.62 m. Ans

Problem 17.50

Continue from Prob. 17.49, if charge q_2 is altered to +10e and both charges are still at the original position, at what position is the resultant electric field equal to zero?

Solution



Consider the position where the resultant electric field can be zero by the same method as Prob. 17.49. We find that the desired position must be on the line passing through both charges, the x–axis. However, in this case, charges q_1 and q_2 both are positive, electric fields $\vec{E}_{q1}$ and $\vec{E}_{q2}$ at points B and C in the above figure have the same direction, and the resultant electric field at these points cannot be zero.

The desired position is point A between the two charges. At this point, electric fields $\vec{E}_{q1}$ and $\vec{E}_{q2}$ have the opposite direction, and point A must be closer to charge q_1 than q_2, causing the magnitudes of electric fields $\vec{E}_{q1}$ and $\vec{E}_{q2}$ able to be the same.

Given point A far away from charge q_1 by distance L. Therefore, point A and charge q_2 are separated by distance d – L. The magnitude of electric fields $\vec{E}_{q1}$ and $\vec{E}_{q2}$ at point A are

$$E_{q1} = k_0 \frac{q_1}{L^2}$$

$$E_{q2} = k_0 \frac{q_2}{(d - L)^2}$$

The resultant electric field at point A equals zero when

$$E_{q1} = E_{q2}$$

$$k_0 \frac{q_1}{L^2} = k_0 \frac{q_2}{(d - L)^2}$$

$$\left(\frac{d - L}{L}\right)^2 = \frac{q_2}{q_1}$$

$$\frac{d}{L} - 1 = \sqrt{\frac{q_2}{q_1}}$$

$$\frac{d}{L} = 1 + \sqrt{\frac{q_2}{q_1}}$$

$$L = \frac{d}{1 + \sqrt{\frac{q_2}{q_1}}}$$

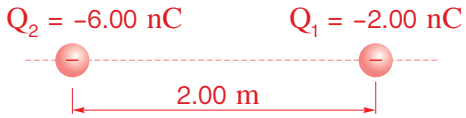
$$= \frac{1.50}{1 + \sqrt{\frac{10e}{5e}}}$$

$$L = 0.621 \text{ m}$$

In conclusion, the position where the resultant electric field equals zero is on the right of charge q_1 by a distance of 0.621 m or at coordinate x = 0.621 m. Ans

Problem 17.51

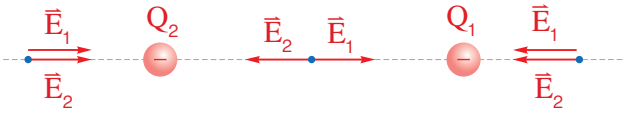
Charges Q_1 = –2.00 nC and Q_2 = –6.00 nC are placed 2.00 m apart, as shown in Fig. (a). Find a position on the line passing through the two charges with a resultant electric field of zero.



(a)

Solution

A resultant electric field at any point is the vector sum of electric fields $\vec{E}_1$ and $\vec{E}_2$ due to charges Q_1 and Q_2, respectively. First, we should begin by considering which parts of the line passing through charges Q_1 and Q_2 can have the resultant electric field equal to zero: on the right of charge Q_1, between the charges, and the last part on the left of charge Q_2, as shown in Fig. (b).

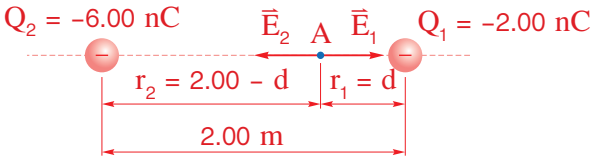


(b)

Since charges Q_1 and Q_2 are negative, each of the electric fields due to the charges at any point is directed towards the charges. Considering the right-hand side of charge Q_1 in Fig. (b), electric fields $\vec{E}_1$ and $\vec{E}_2$ are in the same direction to the left. So, the resultant electric field in this part has no chance to be zero.

Similarly, on the left-hand side of charge Q_2, electric fields $\vec{E}_1$ and $\vec{E}_2$ have the same direction to the right. Thus, this part also has no chance to have the resultant electric field equal to zero.

However, between charges Q_1 and Q_2, electric fields $\vec{E}_1$ and $\vec{E}_2$ have the opposite direction. Therefore, in this part, the resultant electric field can be zero if the magnitudes of $\vec{E}_1$ and $\vec{E}_2$ are the same. Use this fact to find the desired position, as shown in Fig. (c).



(c)

In Fig. (c), given point A being the position where the resultant electric field is zero. Point A is far away from charge Q_1 by distance d. Therefore, it is separated from charge Q_2 by a distance of 2.00 – d. At point A, electric fields $\vec{E}_1$ and $\vec{E}_2$ must be equal in magnitude and opposite in direction to have the resultant electric field equal to zero. Use $E_1 = E_2$ to find distance d, as follows:

$$E_1 = E_2$$

$$k_0 \frac{|Q_1|}{r_1^2} = k_0 \frac{|Q_2|}{r_2^2}$$

$$\frac{|Q_1|}{d^2} = \frac{|Q_2|}{(2.00 - d)^2}$$

$$\left(\frac{2.00 - d}{d}\right)^2 = \frac{|Q_2|}{|Q_1|}$$

$$\frac{2.00 - d}{d} = \sqrt{\frac{|Q_2|}{|Q_1|}}\ , \text{(only positive value)}$$

$$d = \frac{2.00}{\sqrt{\frac{|Q_2|}{|Q_1|}} + 1}$$

$$= \frac{2.00}{\sqrt{\frac{|-6.00 \times 10^{-9}|}{|-2.00 \times 10^{-9}|}} + 1}$$

$$d = 0.732 \text{ m}$$

From the result, it can be concluded that the position where the resultant electric field is zero is between the charges, separated from charge Q_1 by a distance of 0.732 m. Ans

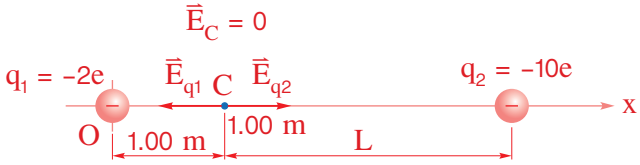
Remarks

There is only a single position where the resultant electric field is equal to zero. It is between the charges and closer to the charge of less magnitude.

Problem 17.52

Charge $q_1 = -2e$ is placed at the origin O, as shown in the figure. What is the position of charge $q_2 = -10e$ that causes the resultant electric field at point C, x = 1.00 m, equal to zero?

Solution



Since charges q_1 and q_2 are negative, applying the conclusion of the previous problem, it is found that point C must be on the line between the charges. It causes charge q_2 placed on the right-hand side of point C, as shown in the figure. Given charge q_2 separated from point C by distance L. Next, find the magnitudes of electric fields $\vec{E}_{q1}$ and $\vec{E}_{q2}$, then set them equal to solve for distance L, which is used to determine the position of charge q_2, as follows:

$$E_{q1} = k_0 \frac{|-2e|}{1.00^2} = k_0(2e)$$

$$E_{q2} = k_0 \frac{|-10e|}{L^2} = k_0 \frac{(10e)}{L^2}$$

The resultant electric field at point C is zero if

$$E_{q1} = E_{q2}$$

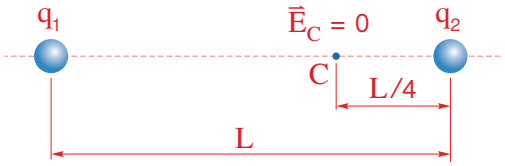
$$k_0(2e) = k_0 \frac{(10e)}{L^2}$$

$$L = \sqrt{5} \text{ m}$$

From the result, it can be concluded that the position of charge q_2 is on the right-hand side of point C by a distance of $\sqrt{5}$ m or at coordinate $x = 1.00 + \sqrt{5} = 3.24$ m. Ans

Problem 17.53

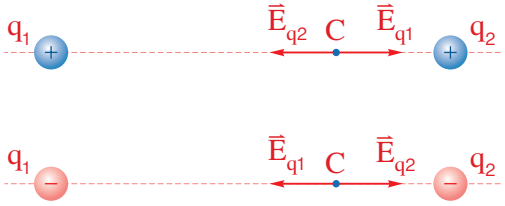
Charges q_1 and q_2 are separated by distance L, as shown in Fig. (a). If the resultant electric field at point C, far away from charge q_2 by distance $\frac{L}{4}$, is zero, find the ratio $\frac{q_1}{q_2}$.



(a)

Solution

Since point C is between charges q_1 and q_2, for electric fields $\vec{E}_{q1}$ and $\vec{E}_{q2}$ at this point being in the opposite direction, as shown in Fig. (b), these two charges must have the same sign, both positive or both negative. However, either positive or negative, the resultant electric field at point C is zero only if the magnitudes of electric fields $\vec{E}_{q1}$ and $\vec{E}_{q2}$ are the same.



(b)

From Fig. (a), point C is separated from charge q_1 by distance $\frac{3L}{4}$, and charge q_2 by distance $\frac{L}{4}$. Therefore, the magnitudes of the electric fields due to the two charges at point C are given by

$$E_{q1} = k_0 \frac{q_1}{\left(\frac{3L}{4}\right)^2}$$

$$E_{q2} = k_0 \frac{q_2}{\left(\frac{L}{4}\right)^2}$$

The resultant electric field at point C is zero when

$$E_{q1} = E_{q2}$$

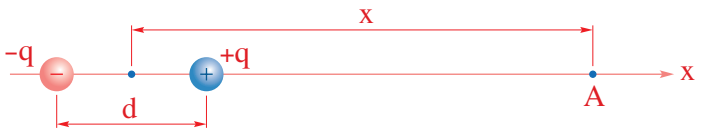
$$k_0 \frac{q_1}{\left(\frac{3L}{4}\right)^2} = k_0 \frac{q_2}{\left(\frac{L}{4}\right)^2}$$

Therefore, the ratio $\frac{q_1}{q_2}$ is

$$\frac{q_1}{q_2} = 3^2 = 9$$ Ans

Problem 17.54

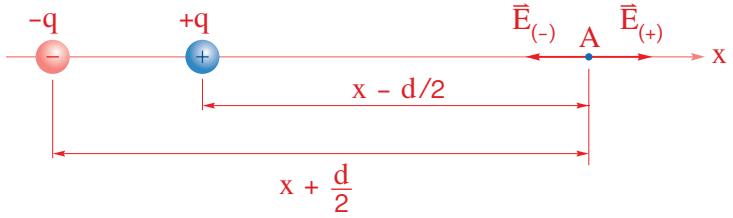
Two point charges of the same magnitude q and opposite signs are placed on the x-axis, a distance d apart, as shown in Fig. (a). Find the resultant electric field at point A on the x-axis, far away from the midpoint of the two charges by distance x, and find this field again when point A is very far from the charges.



(a)

Solution

The resultant electric field at point A is the vector sum of electric fields $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ due to charges +q and –q, as shown in Fig. (b).



(b)

From Fig. (b), electric field $\vec{E}_{(+)}$ is directed outward from charge +q along the positive x-axis, and point A and this charge are separated by distance $x - \frac{d}{2}$. Similarly, electric field $\vec{E}_{(-)}$ is directed towards charge –q along the negative x-axis, and point A and this charge are separated by distance $x + \frac{d}{2}$. So, electric fields $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ are given by

$$\vec{E}_{(+)} = k_0 \frac{|+q|}{\left(x - \frac{d}{2}\right)^2} \vec{i}$$

$$\vec{E}_{(+)} = \frac{k_0 q}{\left(x - \frac{d}{2}\right)^2} \vec{i} \quad \text{... (1)}$$

$$\vec{E}_{(-)} = k_0 \frac{|-q|}{\left(x + \frac{d}{2}\right)^2} (-\vec{i})$$

$$\vec{E}_{(-)} = \frac{-k_0 q}{\left(x + \frac{d}{2}\right)^2} \vec{i} \quad \text{... (2)}$$

When electric fields $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ at point A are known, we can find the resultant electric field at this point, as follows:

$$\vec{E}_A = \vec{E}_{(+)} + \vec{E}_{(-)} \quad \text{... (3)}$$

Substitute $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ from Eqs. (1) and (2) into Eq. (3), we get

$$\vec{E}_A = \frac{k_0 q}{\left(x - \frac{d}{2}\right)^2}\vec{i} + \left(\frac{-k_0 q}{\left(x + \frac{d}{2}\right)^2}\vec{i}\right)$$

$$= k_0 q\left(\frac{1}{\left(x - \frac{d}{2}\right)^2} - \frac{1}{\left(x + \frac{d}{2}\right)^2}\right)\vec{i}$$

$$= k_0 q\left(\frac{\left(x + \frac{d}{2}\right)^2 - \left(x - \frac{d}{2}\right)^2}{\left(x - \frac{d}{2}\right)^2\left(x + \frac{d}{2}\right)^2}\right)\vec{i}$$

$$= k_0 q\left(\frac{\left(x + \frac{d}{2} - x + \frac{d}{2}\right)\left(x + \frac{d}{2} + x - \frac{d}{2}\right)}{\left(x - \frac{d}{2}\right)^2\left(x + \frac{d}{2}\right)^2}\right)\vec{i}$$

$$= k_0 q\frac{d(2x)}{\left(x - \frac{d}{2}\right)^2\left(x + \frac{d}{2}\right)^2}\vec{i}$$

$$\vec{E}_A = \frac{2qdxk_0}{\left(x - \frac{d}{2}\right)^2\left(x + \frac{d}{2}\right)^2}\vec{i} \quad \text{Ans}$$

When point A is very far from the charges, distance x is much larger than separation d. Thus, we can approximate $x - \frac{d}{2} \simeq x$ and $x + \frac{d}{2} \simeq x$, and reduce the expression for resultant electric field $\vec{E}_A$ as

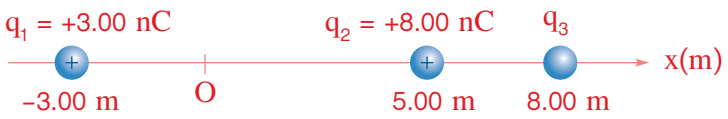
$$\vec{E}_A = \frac{2qdxk_0}{\left(x - \frac{d}{2}\right)^2\left(x + \frac{d}{2}\right)^2}\vec{i}$$

$$\simeq \frac{2qdxk_0}{(x^2)(x^2)}\vec{i}, \quad x >> d$$

$$\vec{E}_A \simeq \frac{2qdk_0}{x^3}\vec{i}, \quad x >> d \qquad \text{Ans}$$

Problem 17.55

Charges q_1 = +3.00 nC, q_2 = +8.00 nC and q_3 are placed on the x-axis, as shown in Fig. (a). What is charge q_3 that causes the electric force on any charge q placed at the origin O equal to zero?



(a)

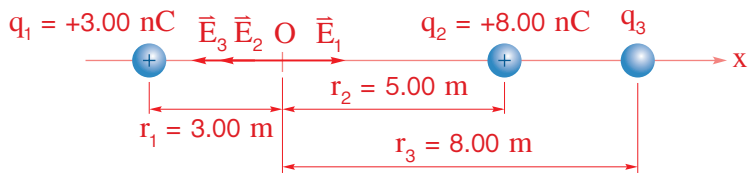
Solution

"The resultant electric force on any charge q at the origin O is zero" means that the resultant electric field at the origin O is zero. So, the problem asks you to find charge q_3 that makes the resultant electric field at the origin O equal to zero.

Resultant electric field $\vec{E}$ at the origin O is the vector sum of electric fields $\vec{E}_1, \vec{E}_2$ and $\vec{E}_3$ due to charges q_1, q_2, and q_3, respectively:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \qquad \ldots (1)$$

Figure (b) shows the data for finding electric field $\vec{E}_1$ and $\vec{E}_2$. Since resultant electric field $\vec{E} = 0$, we can solve for electric field $\vec{E}_3$ by substituting $\vec{E}_1, \vec{E}_2$ and $\vec{E} = 0$ into Eq. (1). Because charge q_3 produces electric field $\vec{E}_3$, the known electric field $\vec{E}_3$ can be used to find the desired charge q_3, as follows:



(b)

Electric field $\vec{E}_1$ is directed outward from charge q_1 along the positive x-axis, therefore

$$\vec{E}_1 = k_0 \frac{|q_1|}{r_1^2} \vec{i} \qquad \text{... (2)}$$

Similarly, electric field $\vec{E}_2$ is directed outward from charge q_2 along the negative x-axis:

$$\vec{E}_2 = k_0 \frac{|q_2|}{r_2^2} (-\vec{i}) \qquad \text{... (3)}$$

Substitute $\vec{E}_1$ and $\vec{E}_2$ from Eqs. (2) and (3) into Eq. (1) and use $\vec{E} = 0$:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$0 = \left(k_0 \frac{|q_1|}{r_1^2} \vec{i} \right) + \left(-k_0 \frac{|q_2|}{r_2^2} \vec{i} \right) + \vec{E}_3$$

$$\vec{E}_3 = k_0 \left(-\frac{|q_1|}{r_1^2} + \frac{|q_2|}{r_2^2} \right) \vec{i}$$

$$= 8.99 \times 10^9 \left(\frac{-|+3.00 \times 10^{-9}|}{3.00^2} + \frac{|+8.00 \times 10^{-9}|}{5.00^2} \right) \vec{i}$$

$$\vec{E}_3 = -\frac{899}{7500} \vec{i} \text{ N/C}$$

Electric field $\vec{E}_3 = -\frac{899}{7500} \vec{i}$ N/C is directed along the negative x-axis. When considering it relative to charge q_3, we see that electric field $\vec{E}_3$ is directed outward

from the charge, so charge q_3 is positive. The magnitude of this charge is obtained by the magnitude of electric field $\vec{E}_3$:

$$E_3 = k_0 \frac{|q_3|}{r_3^2}$$

$$|q_3| = \frac{E_3 r_3^2}{k_0}$$

$$= \frac{\frac{899}{7500} \times 8.00^2}{8.99 \times 10^9}$$

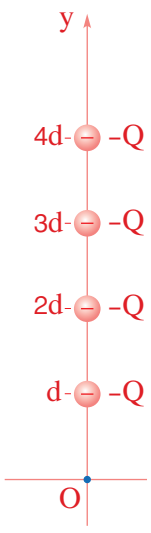
$$= 0.853 \times 10^{-9}\ \text{C}$$

$$|q_3| = 0.853\ \text{nC}$$

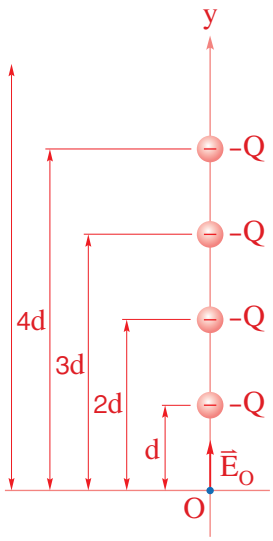
Since q_3 is positive, it can be concluded that $q_3 = +0.853$ nC. Ans

Problem 17.56

An infinite number of identical charges –Q are placed on the y–axis at positions y = d, 2d, 3d, ..., as shown in Fig. (a). Determine the electric field due to these charges at the origin O. Given $\sum_{i=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, where n is a positive integer.



(a)



(b)

Solution

The resultant electric field at the origin O is the infinite vector sum of the electric fields due to all the charges –Q at positions y = d, 2d, 3d, ... The electric field due to each charge is directed towards that charge along the positive y–axis. So, the resultant electric field $\vec{E}_O$ at the origin O is also in the positive y–direction, as shown in Fig. (b).

Since the electric field at the origin O due to each charge is in the same direction, the magnitude of the resultant electric field at the origin O is the algebraic sum of all the magnitude of the electric fields due to each charge. Charges –Q are separated from the origin O by distances d, 2d, 3d, 4d, ... to infinity, as shown in Fig. (B), therefore, the magnitude of the resultant electric field can be found, as follows:

$$E_O = E_1 + E_2 + E_3 + \ldots$$

$$= k_0 \frac{|-Q|}{d^2} + k_0 \frac{|-Q|}{(2d)^2} + k_0 \frac{|-Q|}{(3d)^2} + \ldots$$

$$= k_0 \frac{Q}{d^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\right)$$

$$E_O = k_0 \frac{Q}{d^2} \sum_{i=1}^{\infty} \frac{1}{n^2}, \text{ n is a positive integer}$$

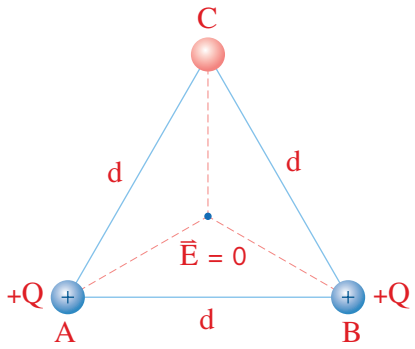
From $\sum_{i=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, the magnitude E_O can be obtained by

$$E_O = k_0 \frac{Q}{d^2}\left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{6}\frac{k_0 Q}{d^2}$$

In conclusion, the electric field at the origin O is $\frac{\pi^2}{6}\frac{k_0 Q}{d^2}\vec{j}$ Ans

Problem 17.57

On an equilateral triangle ABC, two point charges +Q are placed at corners A and B, as shown in Fig. (a). At corner C, there is an unknown charge. If the resultant electric field at the center of the triangle is zero, find the charge at corner C.



(a)

Solution

Resultant electric field $\vec{E}$ at the center of the triangle is the vector sum of the electric fields due to the charges at points A, B, and C:

$$\vec{E} = \vec{E}_A + \vec{E}_B + \vec{E}_C$$

The problem gives the resultant electric field $\vec{E} = 0$:

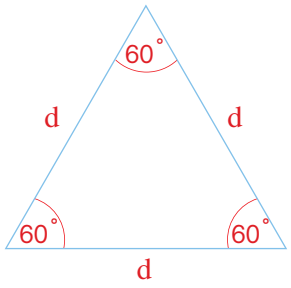
$$0 = \vec{E}_A + \vec{E}_B + \vec{E}_C \quad \text{... (1)}$$

Steps for finding the charge at point C: begin by calculating electric fields $\vec{E}_A$ and $\vec{E}_B$ at the center of the triangle from the known charges at points A and B, then substitute them into Eq. (1) to solve for electric field $\vec{E}_C$, and finally use $\vec{E}_C$ to determine the charge at center C.

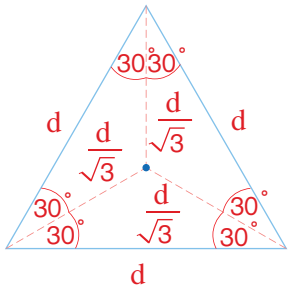
Here, we would represent electric fields $\vec{E}_A, \vec{E}_B$, and $\vec{E}_C$, including the expression in Eq. (1) by drawing arrows for the electric fields. It gives a variety of ways to solve problems.

First of all, we will discuss the properties of an equilateral triangle needed for this problem:

- Each internal angle of an equilateral triangle is 60°, as shown in Fig. (b).
- A line drawn from a corner to the center of an equilateral triangle has a length of $\frac{d}{\sqrt{3}}$, where d is the triangle's side length, as in Fig. (c). Also, this line bisects the internal angle into two equal angles, each of 30°.

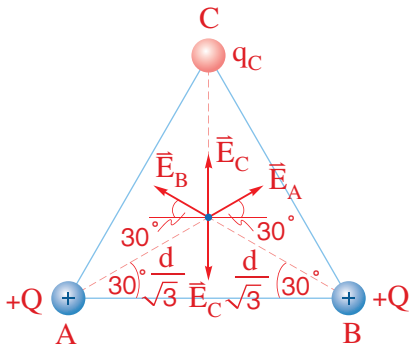


(b)



(c)

Begin by finding electric fields $\vec{E}_A$ and $\vec{E}_B$ at the center of the triangle, as in Fig. (d). From the figure, electric field $\vec{E}_A$ is directed outward from point A, making a 30° angle with the horizontal to the right of the center. Similarly, electric fields $\vec{E}_B$ is directed outward from point B, making a 30° angle with the horizontal to the left of the center.



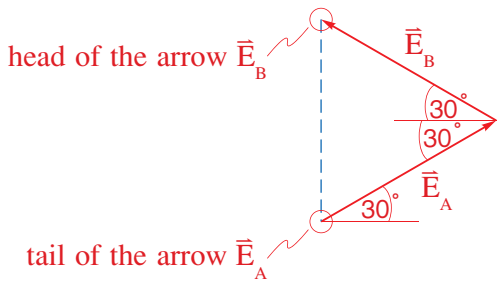
(d)

For the magnitudes of electric fields $\vec{E}_A$ and $\vec{E}_B$, we see that they are the same because the charges at points A and B have the same magnitude Q, and both points are equidistant from the center by distance $\frac{d}{\sqrt{3}}$. Therefore, we draw the arrows for electric fields $\vec{E}_A$ and $\vec{E}_B$ with the same length.

In Fig. (d), an important thing that we can tell about electric field $\vec{E}_C$ is that its direction is vertical but cannot indicate whether directed upward to the charge at point C or downward out from the charge because the charge's type is unknown. We also cannot determine the magnitude of electric field $\vec{E}_C$ since the charge's magnitude is unknown, although distance $\frac{d}{\sqrt{3}}$ from point C to the center is known. However, all the data of electric fields $\vec{E}_A, \vec{E}_B$, and $\vec{E}_C$, as mentioned, can be used together with Eq. (1) to solve for electric field $\vec{E}_C$, as follows:

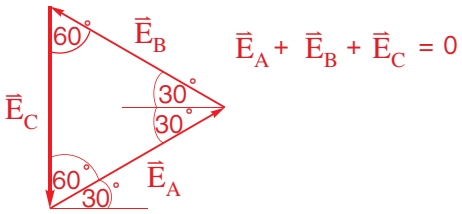
From Eq. (1), $\vec{E}_A + \vec{E}_B + \vec{E}_C = 0$ means that we have to add them by joining each arrow together in a head-to-tail form. Then, "the vector sum of the three electric fields equals zero" refers to the head of the last vector being at the tail of the first vector, so they form a closed triangle.

In Fig. (e), draw an arrow for electric field $\vec{E}_A$ making a 30° angle above the horizontal to the right, then sketch an arrow for electric field $\vec{E}_B$ making a 30° angle above the horizontal to the left with the arrowhead of $\vec{E}_A$ connecting to the arrow tail of $\vec{E}_B$. Therefore, the angle between $\vec{E}_A$ and $\vec{E}_B$ is $30^\circ + 30^\circ = 60^\circ$, as shown in the figure. Since the two electric fields have the same magnitude, the arrows of electric fields $\vec{E}_A$ and $\vec{E}_B$ have equal length, causing the head of the arrow $\vec{E}_B$ aligned vertically with the tail of the arrow $\vec{E}_A$.



(e)

Next, we would draw an arrow for electric field $\vec{E}_C$ by sketching it from the arrowhead of $\vec{E}_B$ to the arrow tail of $\vec{E}_A$ to form a closed triangle, which means the vector sum of the three electric fields is equal to zero, as shown in Fig. (f). From the figure, when the arrow was drawn, we see that electric field $\vec{E}_C$ is directed downward, compared to Fig. (d) it means electric field $\vec{E}_C$ is directed outward from the charge at point C, so the charge at point C is positive.



(f)

Furthermore, the angle of 60° between the arrows of $\vec{E}_A$ and $\vec{E}_B$ and the same length of these two arrows make the other internal angles of the triangle having the same size equal to $\frac{180^\circ - 60^\circ}{2} = 60^\circ$. We find that all the internal angles of the triangle in Fig. (f) have the same size of 60°, so it is an equilateral triangle. This gives us that the arrows of $\vec{E}_A, \vec{E}_B$ and $\vec{E}_C$ have the same length:

$$E_A = E_B = E_C$$

Since in Fig. (a) point C is at the same distance from the triangle's center as points A and B, the magnitude of the charge at point C must also be equal to that of the charges at points A and B, the magnitude Q. Previously, we have known that the charge at point C is positive, so it is +Q, namely, the charge at every corner is +Q. Ans

"Electric Field due to continuously distributed Charge"

Problem 17.58

A charge of 9.00 nC is distributed uniformly on the following objects. Find the charge density.

1) a straight line of length 2.00 m

2) a circular arc of radius 2.00 m, subtending an angle of 30° at its center

3) a surface of a circular disc of radius 2.00 m

4) the surface of a sphere of radius 2.00 m

5) the volume of a sphere of radius 2.00 m

Solution

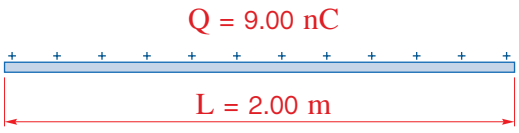
1) For charge $Q = 9.00 \times 10^{-9}$ C distributed uniformly on a straight line of length L = 2.00 m, as shown in Fig. (a), its linear charge density is

$$\lambda = \frac{Q}{L}$$

$$= \frac{9.00 \times 10^{-9}\ \text{C}}{2.00\ \text{m}}$$

$$= 4.50 \times 10^{-9}\ \text{C/m}$$

$$\lambda = 4.50\ \text{nC/m}$$ Ans



(a)

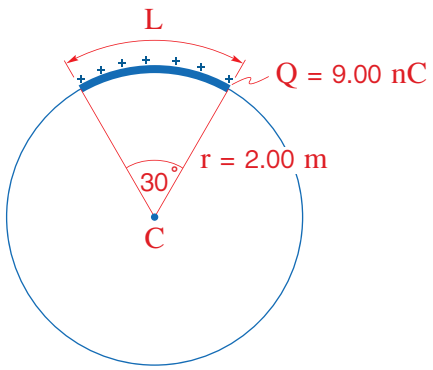
2) A circular arc of radius r = 2.00 m, subtending an angle of 30° at its center C, as shown in Fig. (b), has length L:

$$\frac{L}{r} = \theta\ (\text{rad})$$

$$L = r\theta\ (\text{rad})$$

$$L = 2.00 \times 30^\circ \times \frac{\pi}{180^\circ}\ m$$

$$L = 2.00 \times \frac{\pi}{6}\ m$$



(b)

Hence, the linear charge density of charge $Q = 9.00 \times 10^{-9}$ C along the circular arc can be obtained, as follows:

$$\lambda = \frac{Q}{L}$$

$$= \frac{9.00 \times 10^{-9}\ C}{2.00 \times \frac{\pi}{6}\ m}$$

$$= 8.59 \times 10^{-9}\ C/m$$

$$\lambda = 8.59\ nC/m \quad \text{Ans}$$

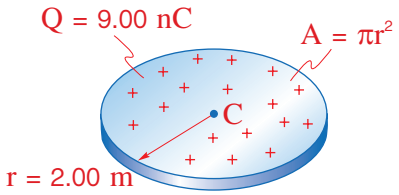
3) One surface of a circular disk with radius r = 2.00 m has area $A = \pi r^2 = \pi \times 2.00^2\ m^2$, as in Fig. (c), so the surface charge density of charge $Q = 9.00 \times 10^{-9}$ C on area A is

$$\sigma = \frac{Q}{A}$$

$$= \frac{9.00 \times 10^{-9}\ C}{\pi \times 2.00^2\ m^2}$$

$$= 0.716 \times 10^{-9}\ C/m^2$$

$$\sigma = 0.716\ nC/m^2 \quad \text{Ans}$$



(c)

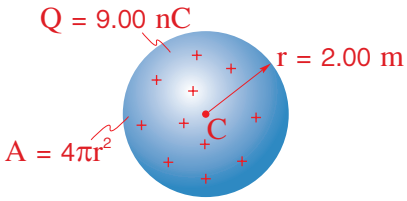
4) A sphere of radius r = 2.00 m has surface area $A = 4\pi r^2 = 4\pi \times 2.00^2\ m^2$, as in Fig. (d), so the surface charge density of charge $Q = 9.00 \times 10^{-9}$ C on area A is

$$\sigma = \frac{Q}{A}$$

$$= \frac{9.00 \times 10^{-9}\ C}{4\pi \times 2.00^2\ m^2}$$

$$= 0.719 \times 10^{-9}\ C/m^2$$

$$\sigma = 0.719\ nC/m^2$$ Ans



distributed on the surface

(d)

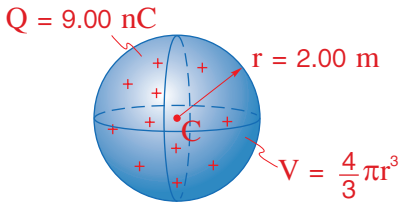
5) A sphere of radius r = 2.00 m has a volume of $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 2.00^3\ m^3$, as in Fig. (e), so the volume charge density of charge $Q = 9.00 \times 10^{-9}$ C through volume V is

$$\rho = \frac{Q}{V}$$

$$= \frac{9.00 \times 10^{-9}\ C}{\frac{4}{3}\pi \times 2.00^3\ m^3}$$

$$= 0.269 \times 10^{-9}\ C/m^3$$

$$\rho = 0.269 \text{ nC/m}^3$$ Ans



distributed through the volume

(e)

Problem 17.59

1) A straight line 8.00 cm long has a uniformly distributed charge with a linear charge density of 6.00 nC/m. Find the total charge on the line.

2) A solid cylinder has a radius of 3.00 cm and a length of 6.00 cm. All the cylinder surfaces have a uniformly distributed charge with a surface charge density of 9.00 nC/m^2. Find the total charge on the cylinder's surfaces.

3) An insulating sphere of radius 3.00 cm has a uniformly distributed charge through its volume with a volume charge density of 15.0 nC/m^3. Find the total charge of the sphere.

Solution

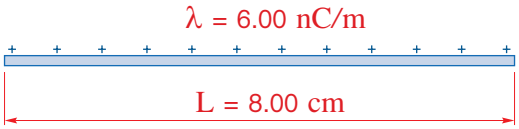
1) We know length L = 8.00 cm = 8.00 × 10^{-2} m and linear charge density λ = 6.00 nC/m = 6.00 × 10^{-9} C/m, as shown in Fig. (a), so total charge Q on the line is given by

$$\lambda = \frac{Q}{L}$$

$$Q = \lambda L$$

$$= 6.00 \times 10^{-9} \times 8.00 \times 10^{-2} \text{ C}$$

$$Q = 4.80 \times 10^{-10} \text{ C}$$ Ans



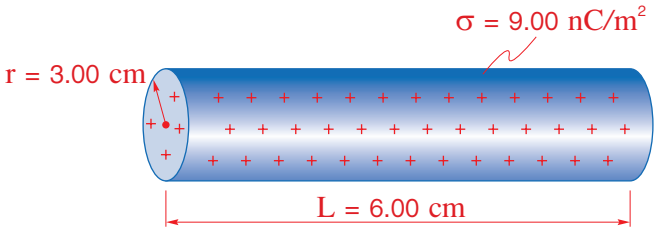
(a)

2) A solid cylinder of radius r = 3.00 cm = 3.00×10^{-2} m and length L = 6.00 cm = 6.00×10^{-2} m has a total surface area:

$$A = \text{two circular areas of radius r + lateral curved area}$$

$$= 2(\pi r^2) + 2\pi rL$$

$$A = 2\pi r(r + L)$$



(b)

The charge is distributed uniformly with the surface charge density $\sigma = 9.00\ nC/m^2 = 9.00 \times 10^{-9}\ C/m^2$, so total charge Q on all the cylinder surfaces is

$$\sigma = \frac{Q}{A}$$

$$Q = \sigma A$$

$$= \sigma \times 2\pi r(r + L)$$

$$= 9.00 \times 10^{-9} \times 2\pi \times 3.00 \times 10^{-2}(3.00 \times 10^{-2} + 6.00 \times 10^{-2})\ C$$

$$Q = 1.53 \times 10^{-10}\ C$$ Ans

3) A sphere of radius r = 3.00 cm = 3.00×10^{-2} m has volume $V = \frac{4}{3}\pi r^3$. Since the volume charge density is $\rho = 15.0\ nC/m^3 = 15.0 \times 10^{-9}\ C/m^3$, the total charge Q of the sphere is given by

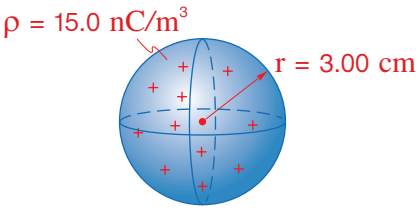
$$\rho = \frac{Q}{V}$$

$$Q = \rho V$$

$$= \rho\left(\frac{4}{3}\pi r^3\right)$$

$$= 15.0 \times 10^{-9} \times \frac{4}{3}\pi \times (3.00 \times 10^{-2})^3 \ \text{C}$$

$$Q = 1.70 \times 10^{-12} \ \text{C}$$ Ans

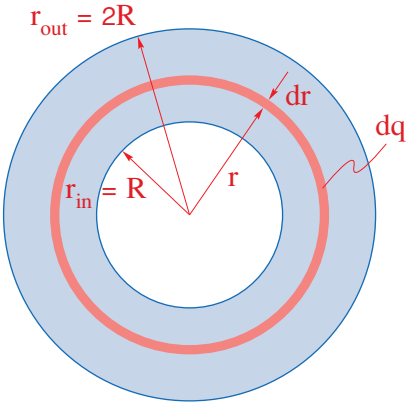


(c)

Problem 17.60

An insulating spherical shell has inner radius R and outer radius 2R. Through its volume, a charge is distributed with a volume charge density varying with radius r measured from the center of the shell, $\rho = kr$ where k is constant. Find the total charge of the shell.

Solution



Since the volume charge density is not constant, increasing with radius r as the equation $\rho = kr$, we begin by choosing an infinitesimal shell-shaped element of radius r and thickness dr, as shown in the figure. Then determine charge dq on this element. Finally, find the total charge on the shell by integrating charge dq with respect to radius r from $r = r_{in} = R$ to $r = r_{out} = 2R$, as follows:

A spherical shell element of radius r and thickness dr has volume $dV = 4\pi r^2\, dr$. Since the volume charge density is $\rho = kr$, charge dq on the element is given by

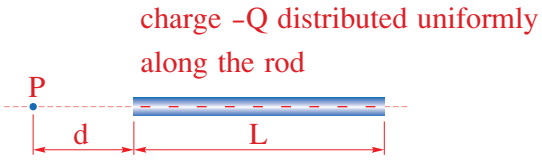
$$\begin{aligned} dq &= \rho\, dV \\ &= kr\,(4\pi r^2\, dr) \\ dq &= 4\pi k r^3\, dr \end{aligned}$$

When charge dq of the element is known, we can find the total charge Q of the spherical shell by calculating the integral of dq from radius $r = r_{in} = R$ to $r = r_{out} = 2R$, as follows:

$$\begin{aligned} Q &= \int dq \\ &= \int_{r_{in}}^{r_{out}} 4\pi k r^3 dr \\ &= 4\pi k \int_{r_{in}}^{r_{out}} r^3 dr \\ &= 4\pi k \left.\frac{r^4}{4}\right|_{r_{in}}^{r_{out}} \\ &= \pi k \left(r_{out}^4 - r_{in}^4\right) \\ &= \pi k \left((2R)^4 - R^4\right) \\ &= \pi k \,(16R^4 - R^4) \\ Q &= 15\pi k R^4 \end{aligned}$$ Ans

Problem 17.61

A rod of length L has charge –Q distributed uniformly along the rod. Find the electric field at point P along the axis of the rod, far away from one end of the rod by distance d, as in Fig. (a).

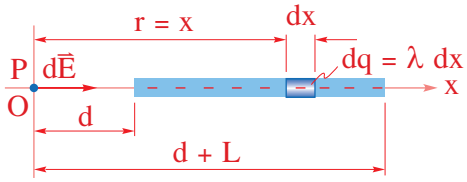


(a)

Solution

In this case, charge –Q is continuously distributed on the rod, not a point charge, so we will find the electric field at point P by the concept of calculus.

Begin by selecting an infinitesimal element of the rod. Then find the electric field at point P due to this element. Finally, calculate the net (resultant) electric field at point P due to charge –Q by integrating the electric field from the element over the entire length of the rod, as follows:



(b)

Let the rod be on the x–axis and point P at the origin O, a distance d from the left end of the rod, as shown in Fig. (b).

Choose an infinitesimal element of tiny length dx, far away from point P or the origin O by distance x, as shown in Fig. (b). It can be seen that this element has charge $dq = \lambda\, dx$, where λ is the linear charge density of the rod equal to $\lambda = -\frac{Q}{L}$, so $dq = -\frac{Q}{L}dx$.

Since charge $dq = -\frac{Q}{L}dx$ is negative, electric field $d\vec{E}$ at point P due to the element is directed towards charge dq along the positive x-axis, as shown in Fig. (b). Charge dq is at distance r = x from point P, therefore, the magnitude of electric field $d\vec{E}$ is given by

$$dE = k_0 \frac{|dq|}{r^2}$$

$$= k_0 \frac{\left|-\frac{Q}{L}dx\right|}{x^2}$$

$$dE = k_0 \frac{Q}{L}\frac{dx}{x^2} \quad \text{... (1)}$$

Since the other elements of the rod produce electric fields at the point P in the positive x-direction, as well, the net electric field, $\vec{E}$, due to all the charge of the rod is also directed along the positive x-axis.

For the magnitude of the net electric field, we can find it by summing all the magnitudes of the electric fields due to every element, which is the integral of dE in Eq. (1) over the rod's length from x = d to x = d + L, as follows:

$$E = \int dE$$

$$= \int_L^{d+L} k_0 \frac{Q}{L}\frac{dx}{x^2}$$

$$= k_0 \frac{Q}{L}\int_L^{d+L} \frac{dx}{x^2}, \quad k_0 \frac{Q}{L} \text{ is constant}$$

$$= k_0 \frac{Q}{L}\left(-\frac{1}{x}\right)\Bigg|_d^{d+L}$$

$$= k_0 \frac{Q}{L}\left(\frac{1}{d} - \frac{1}{d+L}\right)$$

$$= k_0 \frac{Q}{L}\left(\frac{d+L-d}{d(d+L)}\right)$$

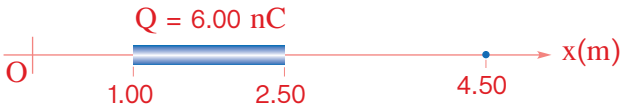
$$E = \frac{k_0 Q}{d(d + L)}$$

In conclusion, the electric field at point P due to charge –Q on the rod of length L has a magnitude of $\frac{k_0 Q}{d(d + L)}$ and is directed along the positive x–axis, or written in vector form as

$$\vec{E} = \frac{k_0 Q}{d(d + L)} \vec{i}$$ Ans

Problem 17.62

A rod lies on the x–axis from x = 1.00 m to x = 2.50 m, as shown in Fig. (a). If charge 6.00 nC is distributed uniformly on the rod, find the electric field at position x = 4.50 m on the x–axis.



(a)

Solution

This problem requires readers to use the expression for an electric field due to a uniformly distributed charge on a rod in the previous problem.

For a charge of magnitude Q distributed uniformly on a rod of length L, a point far away from one end of the rod by distance d along the rod's axis, has an electric field directed outward from the rod if the charge is positive, on the other hand, if negative, the electric field is directed toward the rod. However, either positive or negative charge, the magnitude of the electric field is given by

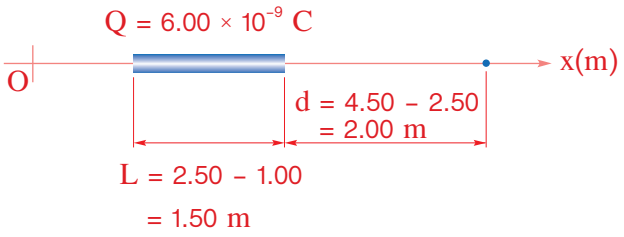
$$E = \frac{k_0 Q}{d(d + L)} \quad \ldots (1)$$

From Fig. (a), we find that $Q = 6.00 \times 10^{-9}$ C, L = 2.50 – 1.00 = 1.50 m, and d = 4.50 – 2.50 = 2.00 m, as shown in Fig. (b). Therefore, the magnitude of the electric

field at position x = 4.50 m is given by Eq. (1), as follows:

$$E = \frac{8.99 \times 10^9 \times 6.00 \times 10^{-9}}{2.00(2.00 + 1.50)}$$

$$E = 7.71 \text{ N/C}$$



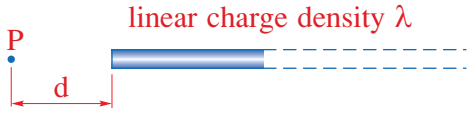
(b)

Since the charge on the rod is positive, the electric field at x = 4.50 m is directed outward from the rod along the positive x-axis. So the electric field at that point can be written in vector form as

$$\vec{E} = 7.71\,\vec{i} \text{ N/C}$$ Ans

Problem 17.63

An infinite straight rod has a uniformly distributed charge along its length with linear charge density λ. Find the electric field at point P, a distance d from the left end of the rod, as shown in Fig. (a).

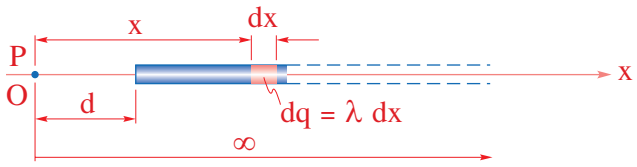


(a)

Solution

Let the rod lie on the x-axis, and point P is at the origin O, as shown in Fig. (b). Consider the direction of the electric field at point P, just as the case of the rod of length L in the past problem. We see that if the rod's charge is positive, which means linear charge density λ is also positive, the electric field at point P will point away

from the rod along the negative x-axis. On the other hand, if the charge is negative (λ is negative), the electric field is directed towards the rod along the positive x-axis.



(b)

Next, find the magnitude of the electric field at point P. Begin by picking up an infinitesimal element of length dx, far away from point P by distance r = x, as shown in Fig. (b). From the figure, the charge on this element is dq = λ dx. So the magnitude of the electric field at point P due to the charge dq is

$$dE = k_0 \frac{|dq|}{r^2}$$

$$= k_0 \frac{|\lambda dx|}{x^2}$$

$$dE = k_0 |\lambda| \frac{dx}{x^2} \qquad \ldots (1)$$

Sincee every element of the rod produces an electric field at point P in the same direction, the magnitude of the net electric field due to all the rod's charge is the integral of dE in Eq. (1) from x = d to x = ∞, as follows:

$$E = \int dE$$

$$= \int_d^\infty k_0 |\lambda| \frac{dx}{x^2}$$

$$= k_0 |\lambda| \int_d^\infty \frac{dx}{x^2}$$

$$= k_0 |\lambda| \left(-\frac{1}{x} \right) \Bigg|_d^\infty$$

$$= k_0 |\lambda| \left(-\frac{1}{\infty} + \frac{1}{d} \right)$$

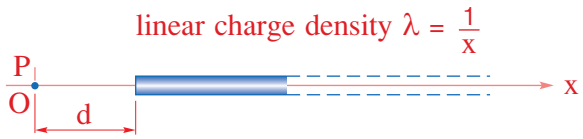
$$= k_0|\lambda|\left(0 + \frac{1}{d}\right)$$

$$E = \frac{k_0|\lambda|}{d}$$

It can be concluded that the electric field at point P has magnitude $\frac{k_0|\lambda|}{d}$ and directed outward from the rod when $\lambda > 0$, and towards the rod when $\lambda < 0$. Ans

Problem 17.64

An infinite straight rod is placed on the x-axis, as shown in Fig. (a). If a charge is distributed on the rod with a linear charge density decreasing as x, $\lambda = \frac{1}{x}$, find the electric field at point P located at the origin O.



(a)

Solution

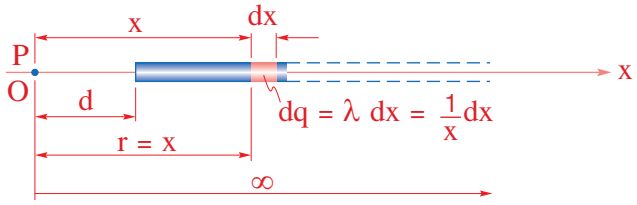
Since the rod is placed on the positive x-axis, linear charge density $\lambda = \frac{1}{x}$ is also positive. It means the charge on the rod is positive. Therefore, the electric field at point P is directed outward from the rod in the negative x-direction.

The magnitude of the electric field at point P can be obtained by choosing an infinitesimal element of length dx, away from point P or the origin O by distance r = x, as shown in Fig. (b). The charge of this element is $dq = \lambda\, dx = \frac{1}{x}dx$. So magnitude dE of the electric field at point P due to the element is given by

$$dE = k_0\frac{|dq|}{r^2}$$

$$= k_0\frac{\left(\frac{1}{x}dx\right)}{x^2}$$

$$dE = k_0 \frac{dx}{x^3} \qquad \ldots (1)$$



(b)

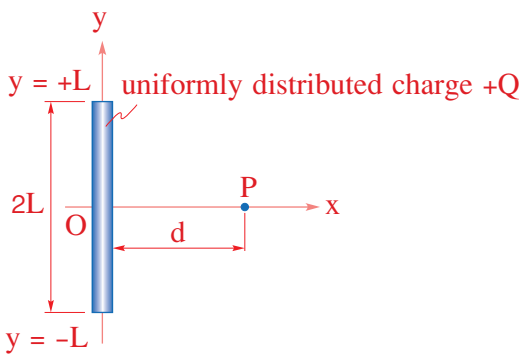
Since every element of the rod produces electric fields in the same negative x-direction, the direction of the net electric field is also in the negative x-direction. Moreover, the magnitude of the net electric field at point P due to all the charge on the rod will be equal to the integral of dE in Eq. (1) from x = d to x = ∞, as follows:

$$\begin{aligned}
E &= \int dE \\
&= \int_d^\infty k_0 \frac{dx}{x^3} \\
&= k_0 \int_d^\infty \frac{dx}{x^3} \\
&= k_0 \left(-\frac{1}{2x^2} \right) \Bigg|_d^\infty \\
&= k_0 \left(-\frac{1}{2 \times \infty^2} + \frac{1}{2d^2} \right) \\
&= k_0 \left(0 + \frac{1}{2d^2} \right) \\
E &= \frac{k_0}{2d^2}
\end{aligned}$$

In conclusion, the electric field at point P has magnitude $\frac{k_0}{2d^2}$ directed along the negative x-axis, or written in vector form as $\vec{E} = -\frac{k_0}{2d^2}\vec{i}$. Ans

Problem 17.65

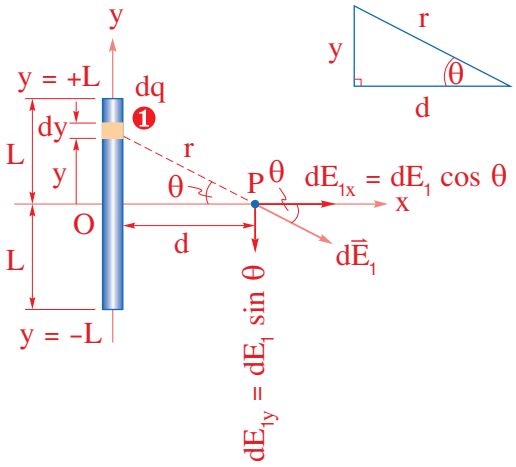
Charge +Q is distributed uniformly on a rod of length 2L placed on the y-axis between positions y = -L and y = +L, as shown in Fig. (a). Determine the electric field at point P on the x-axis, a distance d from the origin O.



(a)

Solution

This problem requires readers to find an electric field due to continuously and uniformly distributed charge along a straight rod at point P on the perpendicular bisector of the rod. The procedure for determining the electric field still uses the concept of calculus, as follows:



(b)

In Fig. (b), we choose an infinitesimal element at point ❶. This element has length dy and is at distance y from the origin O. Since the charge on the rod is positive, electric field $d\vec{E}_1$ due to charge dq on the element is directed outward from the charge with an angle θ below the positive x-axis, as shown in Fig. (b).

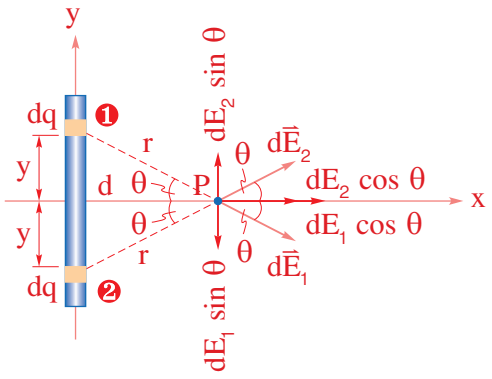
Charge dq at point ❶ is obtained by $dq = \lambda\, dy$. In this case, charge +Q is distributed uniformly along the rod of length 2L, so $\lambda = \frac{+Q}{2L}$ and $dq = \lambda\, dy = \frac{+Q}{2L}\, dy$. Since charge dq and point P are separated by distance r, we can calculate magnitude dE_1 of the electric field due to the charge dq, as the following:

$$dE_1 = k_0 \frac{|dq|}{r^2}$$

$$= k_0 \frac{\left|\frac{+Q}{2L} dy\right|}{r^2}$$

$$dE_1 = \frac{k_0 Q}{2L} \frac{dy}{r^2} \qquad \ldots (1)$$

Because point P is on the x-axis that is the perpendicular bisector of the rod, there is another charge dq at point ❷, which is at distance y from the origin O, just as charge dq at point ❶, but this charge is below the origin O, as shown in Fig. (c). Charge dq at point ❷ and point P are separated by the same distance r as charge dq at point ❶. Thus, electric field $d\vec{E}_2$ at point P due to charge dq at point ❷ has the same magnitude as electric field $d\vec{E}_1$ ($dE_1 = dE_2$) and is directed outward from the charge as well, but with angle θ above the positive x-axis.



(c)

When resolving electric fields $d\vec{E}_1$ and $d\vec{E}_2$ in Fig. (c) into the x- and y-components, it is found that, along the y-axis, they consist of two components of magnitudes $dE_1 \sin\theta$ and $dE_2 \sin\theta$ in the opposite direction. Since $dE_1 = dE_2$, the y-components cancel out to zero, making the net electric field at point P due to charges dq at points ❶ and ❷ having only the x-component.

For charges at other points on the rod, electric fields due to them have the same pattern as the previous charges dq. That is, on the upper and lower parts of the rod from the origin O, there are two equal charges far away from the origin O by the same distance, causing the net electric field at point P due to these two charges having only the x-component. Therefore, we will next use this conclusion to find the net electric field at point P due to all the charges on the rod.

According to Eq. (1), charge dq separated from point P by distance r produces an electric field at point P that has a magnitude

$$dE = \frac{k_0 Q}{2L}\frac{dy}{r^2}$$

Considering only the x-component of the electric field, we get

$$dE_x = dE\cos\theta = \frac{k_0 Q}{2L}\frac{dy}{r^2}\cos\theta \qquad \text{... (2)}$$

In Fig. (b), the right triangle gives

$$\cos\theta = \frac{d}{r}$$

and

$$r = (y^2 + d^2)^{\frac{1}{2}}$$

Substituting the values of $\cos\theta$ and r into Eq. (2) leads to

$$dE_x = \frac{k_0 Q}{2L}\frac{dy}{r^2}\left(\frac{d}{r}\right)$$

$$= \frac{k_0 Qd}{2L}\frac{dy}{r^3}$$

$$dE_x = \frac{k_0 Qd}{2L}\frac{dy}{(y^2 + d^2)^{\frac{3}{2}}} \qquad \text{... (3)}$$

Since the magnitude of the net electric field at point P is the sum of the magnitude dE_x due to the charges on each part of the rod, it is obtained by calculating the integral of dE_x in Eq. (3) with respect to y from y = –L to y = +L:

$$\begin{aligned} E &= E_x \\ &= \int dE_x \\ &= \int_{-L}^{+L} \frac{k_0 Q d}{2L} \frac{dy}{(y^2 + d^2)^{\frac{3}{2}}} \\ &= \frac{k_0 Q d}{2L} \int_{-L}^{+L} \frac{dy}{(y^2 + d^2)^{\frac{3}{2}}}, \quad \frac{k_0 Q d}{2L} \text{ is constant} \end{aligned}$$

The above integral can be found by using the general formula that readers can review in calculus:

$$\int \frac{dy}{(y^2 + d^2)^{\frac{3}{2}}} = \frac{y}{d^2\sqrt{y^2 + d^2}}$$

Substitute the result of the integral into the expression for magnitude E of the electric field:

$$E = \frac{k_0 Q d}{2L} \left(\frac{y}{d^2\sqrt{y^2 + d^2}} \right) \Bigg|_{-L}^{+L}$$

$$= \frac{k_0 Q d}{2L} \left(\frac{2L}{d^2\sqrt{L^2 + d^2}} \right)$$

$$E = \frac{k_0 Q}{d\sqrt{L^2 + d^2}}$$

In conclusion, the net electric field at point P has a magnitude of $\frac{k_0 Q}{d\sqrt{L^2 + d^2}}$ directed along the positive x–axis, or written in vector form as

$$\vec{E} = \frac{k_0 Q}{d\sqrt{L^2 + d^2}} \vec{i}$$ Ans

Remarks

1) If point P is very far from the rod compared to length 2L of the rod, then d >> L. So we can approximate $L^2 + d^2 \simeq d^2$, and the electric field at point P can be approximated as

$$\vec{E} = \frac{k_0 Q}{d\sqrt{d^2}}\vec{i} = \frac{k_0 Q}{d^2}\vec{i}$$

The approximated expression for $\vec{E}$ shows that when point P is very far from the rod, the electric field at point P is equal to that of a point charge with charge +Q separated from point P by distance d. It is well consistent with an observer very far from the rod because it looks like a point.

2) Electric field $\vec{E}$ can be written in linear charge density λ by using $\lambda = \frac{Q}{2L}$ or $Q = 2\lambda L$, as follows:

$$\vec{E} = \frac{k_0(2\lambda L)}{d\sqrt{L^2 + d^2}}\vec{i} = \frac{2k_0\lambda}{d\sqrt{1 + \left(\frac{d}{L}\right)^2}}\vec{i} \qquad \text{... (4)}$$

If making the rod longer and longer, simultaneously adding more charge on it so that linear charge density λ has not changed, then the rod has infinite length and constant linear charge density λ. The electric field due to this infinite rod at a point distance d from the rod can be found by using Eq. (4): set length L so much greater than distance d (L >> d) that $\left(\frac{d}{L}\right)^2 \simeq 0$, therefore,

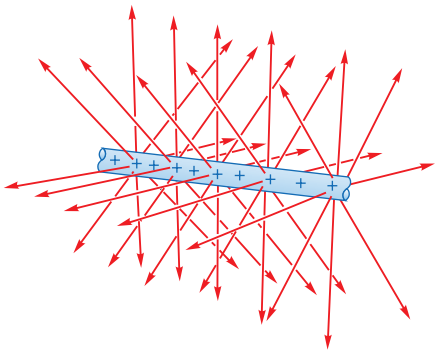
$$\vec{E} = \frac{2k_0\lambda}{d\sqrt{1 + 0}}\vec{i} = \frac{2k_0\lambda}{d}\vec{i}$$

The result indicates that the magnitude of the electric field is $\frac{2k_0\lambda}{d}$. It depends only on distance d from point P to the rod. Therefore, regardless of the direction of point P around the rod, if point P is away in the perpendicular direction from the rod by distance r, the electric field's magnitude is given by

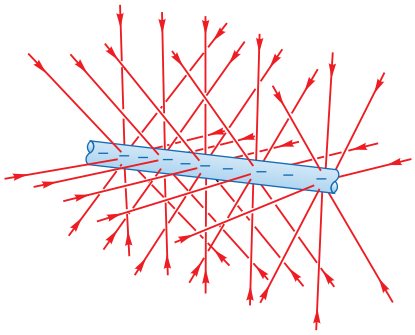
$$E = \frac{2k_0\lambda}{r}, \text{ infinite rod (wire)}$$

In conclusion, the electric field due to an infinite charged rod or wire is proportional to $\frac{1}{r}$, different from the case of a point charge that is proportional to $\frac{1}{r^2}$. For the direction of the electric field, it points radially outward from the rod when the charge on the rod is positive, as shown in Fig. (d), and radially towards the rod when the charge is negative, as in Fig. (e).

(d)

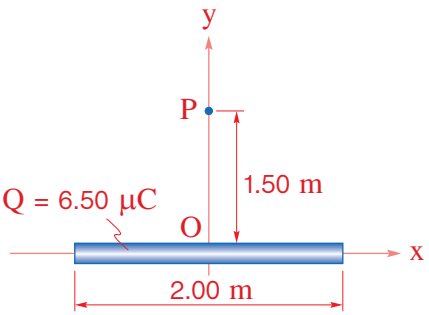


(e)



Problem 17.66

Figure (a) shows a wire of length 2.00 m and charge Q = 6.50 μC distributed uniformly along its length. Find the electric field at point P on the perpendicular bisector of the wire, 1.50 m away from the wire.



(a)

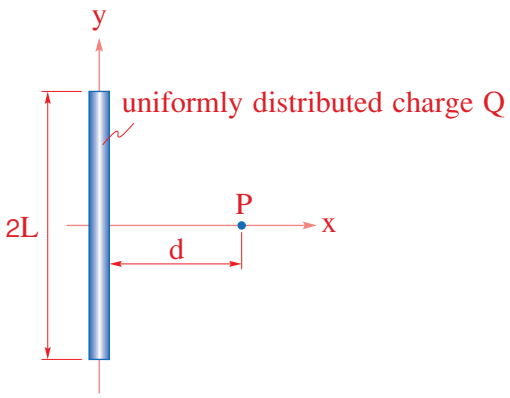
Solution

This problem requires readers to apply the expression for an electric field at a point on the perpendicular bisector of the wire (rod) with a uniformly distributed charge.

From Prob. 17.65, the conclusion is if a wire has length 2L, charge Q distributed uniformly on the wire, and point P is on the perpendicular bisector of the wire distance d from the wire, as shown in Fig. (b), then the magnitude of the electric field at point P is

$$E = \frac{k_0 |Q|}{d\sqrt{L^2 + d^2}} \qquad \text{... (1)}$$

In Fig. (b), if charge Q is positive, the electric field at point P is directed outward from the wire along the positive x-axis. On the other hand, when the charge is negative, the electric field at point P points towards the wire along the negative x-axis.



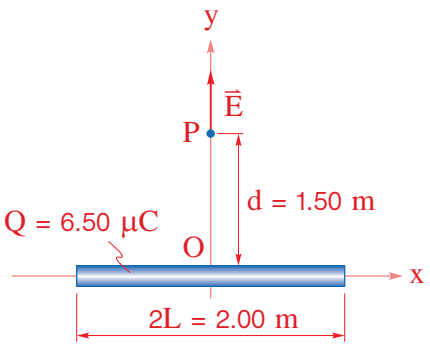
(b)

Apply the conclusion to the wire in Fig. (a), as shown in Fig. (c). The wire has a length of 2L = 2.00 m, that is, L = 1.00 m. Point P and the wire are separated by distance d = 1.50 m. Since charge Q = 6.50 × 10^{-6} C is positive, electric field $\vec{E}$ at point P is directed away from the wire along the positive y-axis and has magnitude

$$E = \frac{k_0 |Q|}{d\sqrt{L^2 + d^2}}$$

$$= \frac{8.99 \times 10^9 \times |6.50 \times 10^{-6}|}{1.50\sqrt{1.00^2 + 1.50^2}}$$

$$E = 2.16 \times 10^4 \text{ N/C}$$



(c)

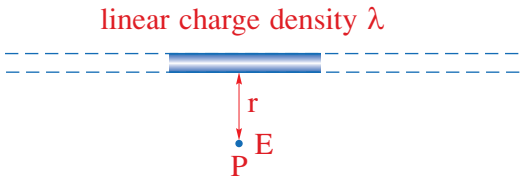
It can be concluded that the electric field at point P has a magnitude of 2.16 × 10^4 N/C directed along the positive y-axis, or written in vector form as

$$\vec{E} = 2.16 \times 10^4 \vec{j} \text{ N/C}$$ Ans

Problem 17.67

An infinite straight wire has a uniformly distributed charge over its length with a linear charge density of 5.50×10^{-8} C/m. Find the distance perpendicular to the wire that the electric field has a magnitude of 1.70×10^{4} N/C.

Solution



From the conclusion of Prob. 17.65, if an infinite straight wire has a uniformly distributed charge along its length with linear charge density λ, as shown in the figure, the electric field at point P perpendicular to the wire by distance r, has magnitude

$$E = \frac{2k_0\lambda}{r}$$

In this case, $\lambda = 5.50 \times 10^{-8}$ C/m and $E = 1.70 \times 10^{4}$ N/C, so distance r is given by

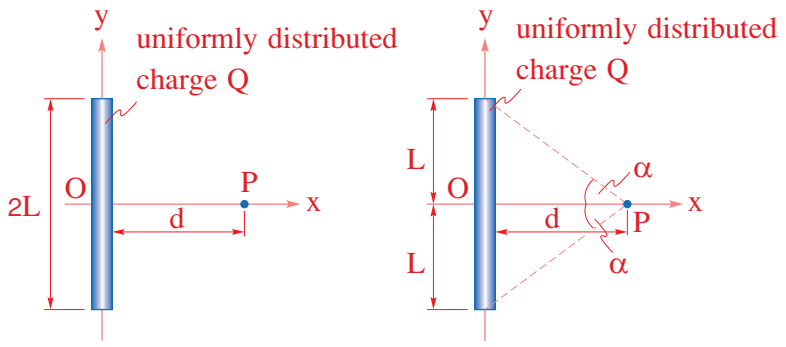
$$r = \frac{2k_0\lambda}{E}$$

$$= \frac{2 \times 8.99 \times 10^{9} \times 5.50 \times 10^{-8}}{1.70 \times 10^{4}}$$

$$r = 0.0582 \text{ m or } 5.82 \text{ cm}$$ Ans

Problem 17.68

From Prob. 17.65, the electric field at point P in Fig. (a) has magnitude $\frac{k_0 Q}{d\sqrt{L^2 + d^2}}$. Transform this expression for the magnitude of the electric field in terms of angle α between the x-axis and the line drawn from point P to the wire's end, as shown in Fig. (b).



(a) (b)

Solution

The electric field at point P has magnitude

$$E = \frac{k_0 Q}{d\sqrt{L^2 + d^2}} \qquad \ldots (1)$$

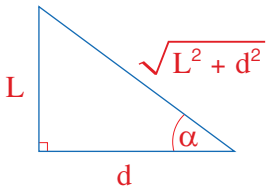
Since charge Q is distributed uniformly along length 2L of the wire, then $\lambda = \frac{Q}{2L}$ and $Q = 2\lambda L$. Substitute $Q = 2\lambda L$ into Eq. (1):

$$E = \frac{k_0 (2\lambda L)}{d\sqrt{L^2 + d^2}}$$

$$E = \frac{2k_0\lambda}{d} \frac{L}{\sqrt{L^2 + d^2}} \qquad \ldots (2)$$

In Fig. (c), the right triangle gives us that

$$\frac{L}{\sqrt{L^2 + d^2}} = \sin \alpha$$



(c)

Substitute the value of $\dfrac{L}{\sqrt{L^2 + d^2}}$ into Eq. (2):

$$E = \frac{2k_0\lambda}{d} \sin \alpha \qquad \text{Ans}$$

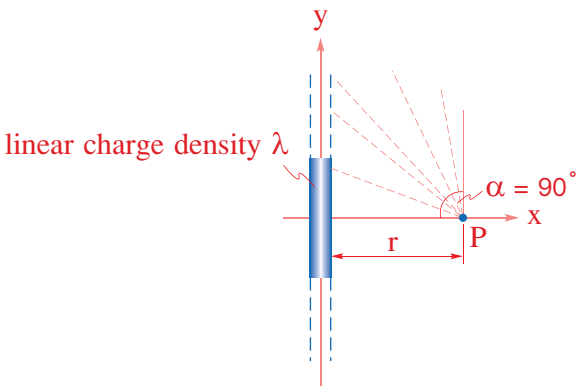
Remarks

If the wire has infinite length, as in Fig. (d), angle $\alpha = 90^\circ$ and the magnitude of the electric field at distance r from the wire is

$$E = \frac{2k_0\lambda}{r} \sin 90^\circ$$

$$E = \frac{2k_0\lambda}{r}, \quad \text{infinite wire}$$

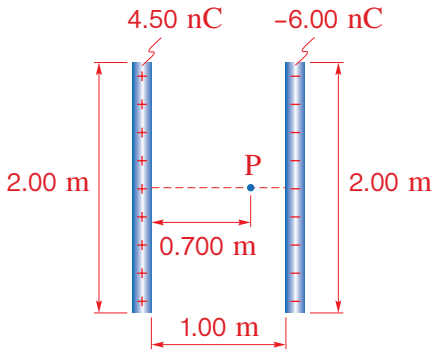
We can see that, in the case of the infinite wire, the magnitude of the electric field is consistent with that of Prob. 17.65.



(d)

Problem 17.69

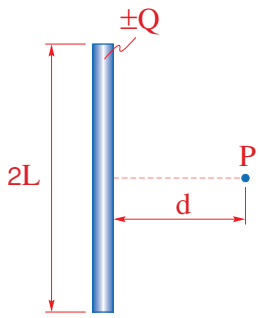
Two straight wires of length 2.00 m lie vertically a distance 1.00 m apart, as shown in Fig. (a). The wire on the left has a charge of 4.50 nC, the wire on the right has a charge of –6.00 nC, and both charges are uniformly distributed. Determine the net electric field at point P on the perpendicular bisector to the wires, far away from the left wire by distance 0.700 m.



(a)

Solution

At point P, the net electric field is the vector sum of the electric fields due to the charges of both wires. So we begin by finding the electric fields at point P due to each wire, then add them vectorially, as follows:



(b)

From Prob. 17.65, the electric field at point P due to a wire of length 2L and charge +Q or –Q distributed uniformly along its length, distance d from the wire, as shown in Fig. (b), has magnitude

$$E = \frac{k_0 Q}{d\sqrt{L^2 + d^2}} \qquad \ldots (1)$$

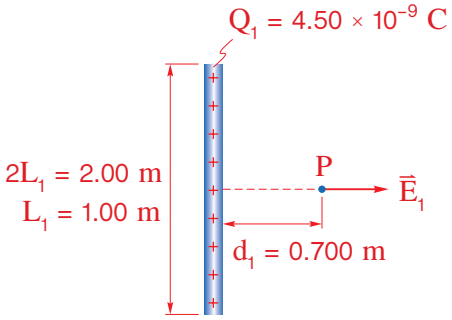
If the charge on the wire is positive, the electric field at point P is directed outward perpendicular to the wire; on the other hand, if negative, directed towards the wire in the perpendicular direction.

For the left wire in Fig. (c), charge Q_1 on the wire is positive. Therefore, it creates electric field $\vec{E}_1$ at point P directed to the right. The magnitude of electric field $\vec{E}_1$ is found by using Eq. (1) and the data in Fig. (c):

$$E_1 = \frac{k_0 Q_1}{d_1\sqrt{L_1^2 + d_1^2}}$$

$$= \frac{8.99 \times 10^9 \times 4.50 \times 10^{-9}}{0.700\sqrt{1.00^2 + 0.700^2}}$$

$$E_1 = 47.3 \text{ N/C}$$



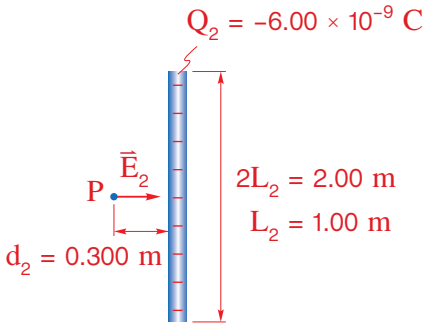
(c)

In the same way, since charge Q_2 is negative, the right wire in Fig. (d) produces electric field $\vec{E}_2$ directed towards the wire. It can be seen that electric fields $\vec{E}_1$ and $\vec{E}_2$ are in the same direction. The magnitude of $\vec{E}_2$ is given by Eq. (1) and the data in Fig. (d):

$$E_2 = \frac{k_0 |Q_2|}{d_2\sqrt{L_2^2 + d_2^2}}$$

$$= \frac{8.99 \times 10^9 \times |-6.00 \times 10^{-9}|}{0.300\sqrt{1.00^2 + 0.300^2}}$$

$$E_1 = 172 \text{ N/C}$$



(d)

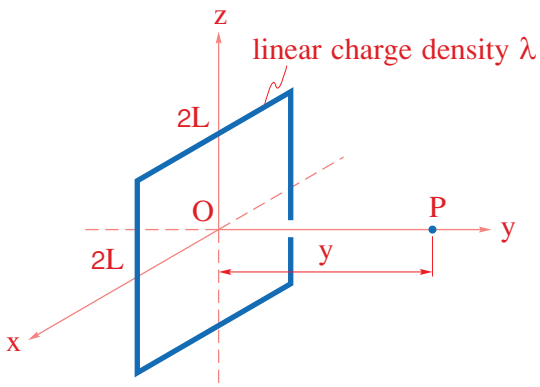
Because electric fields $\vec{E}_1$ and $\vec{E}_2$ are in the same direction to the right, the net electric field at point P is also directed to the right and has a magnitude equal to the sum of magnitudes E_1 and E_2, as follows:

$$E_P = E_1 + E_2 = 47.3 + 172 = 219 \text{ N/C}$$

In conclusion, the net electric field at point P is 219 N/C directed to the right. Ans

Problem 17.70

A square wire of side length 2L has a uniformly distributed charge along its perimeter with linear charge density λ ($\lambda > 0$), as shown in Fig. (a). Find the electric field at the point P on the central axis (the y-axis) of the square, far away from the center O of the square by distance y.

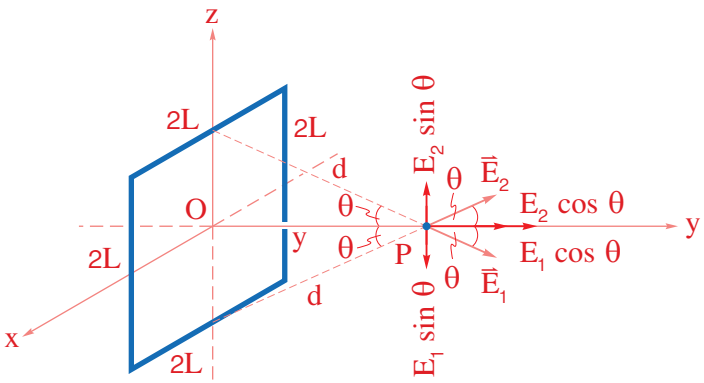


(a)

Solution

Steps for solving: divide the square wire into four straight wires of length 2L, then apply the expression of Prob. 17.65 to find electric fields due to each straight wire, finally add all the electric fields from the four wires vectorially to get the net electric field.

In Fig. (b), the top and bottom straight wires of length 2L are separated from point P by the same distance d. Thus, electric fields $\vec{E}_1$ and $\vec{E}_2$ due to these two wires have the same magnitude ($E_1 = E_2$) and are directed outward from each wire. However, the directions of $\vec{E}_1$ and $\vec{E}_2$ are not the same: at point P, the direction of $\vec{E}_1$ makes an angle θ below the positive y-axis, whereas the direction of $\vec{E}_2$ makes the same angle θ with the positive y-axis, but above the axis, as shown in Fig. (b).



(b)

Therefore, when resolving electric fields $\vec{E}_1$ and $\vec{E}_2$ into their horizontal and vertical components, it is found that, in the vertical, they have two components of equal magnitudes ($E_1 \sin\theta = E_2 \sin\theta$) but opposite directions, making the sum of all the vertical components equal to zero. The remaining are the horizontal components, consisting of two vectors with equal magnitude ($E_1 \cos\theta = E_2 \cos\theta$) and directed along the same positive y-axis.

For another two straight wires lying vertically, they produce electric fields at point P in the same way as the past two wires. Thus, the net electric field at point P is directed along the positive y-axis, and its magnitude equals the sum of the y-components of the electric fields due to the four straight wires. Given the magnitude of the electric field from each wire equal to E, therefore, the net electric field's magnitude is given by

$$E_P = 4E\cos\theta \qquad \text{... (1)}$$

From Prob. 17.65, the electric field at point P due to a straight wire of length 2L and far away from the wire by distance d has magnitude

$$E = \frac{k_0 Q}{d\sqrt{L^2 + d^2}} \qquad \text{... (2)}$$

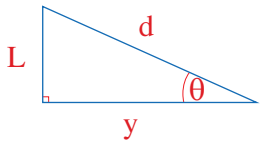
We can find charge Q on the wire of length 2L that has linear charge density λ by using $Q = (2L)\lambda$. Substitute $Q = (2L)\lambda$ into Eq. (2):

$$E = \frac{k_0(2\lambda L)}{d\sqrt{L^2 + d^2}} \qquad \text{... (3)}$$

Figure (c) shows the right triangle of angle θ. We get

$$\cos\theta = \frac{y}{d} \qquad \text{... (4)}$$

$$d^2 = L^2 + y^2 \qquad \text{... (5)}$$



(c)

Substitute E from Eq. (3), and $\cos\theta$ and d^2 from Eqs. (4) and (5) into Eq. (1) to find the magnitude of the net electric field at point P, as follows:

$$E_P = 4\left(\frac{k_0(2L\lambda)}{d\sqrt{L^2 + d^2}}\right)\frac{y}{d}$$

$$= \frac{8k_0 L\lambda y}{d^2\sqrt{L^2 + d^2}}$$

$$= \frac{8k_0 L\lambda y}{(L^2 + y^2)\sqrt{L^2 + L^2 + y^2}}$$

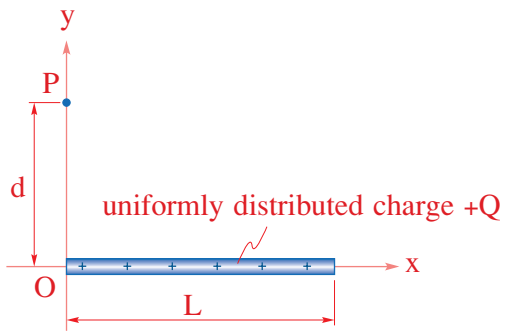
$$E_P = \frac{8k_0L\lambda y}{(L^2 + y^2)\sqrt{2L^2 + y^2}}$$

In conclusion, the net electric field at point P has magnitude $\frac{8k_0L\lambda y}{(L^2 + y^2)\sqrt{2L^2 + y^2}}$ in the positive y-direction, or written in vector form as

$$\vec{E}_P = \frac{8k_0L\lambda y}{(L^2 + y^2)\sqrt{2L^2 + y^2}}\vec{j}$$ Ans

Problem 17.71

A straight wire of length L has uniformly distributed charge +Q along its length and lie on the x-axis, as shown in Fig. (a). Determine the electric field at point P on the y-axis, far away from one end of the wire at the origin O by distance d.

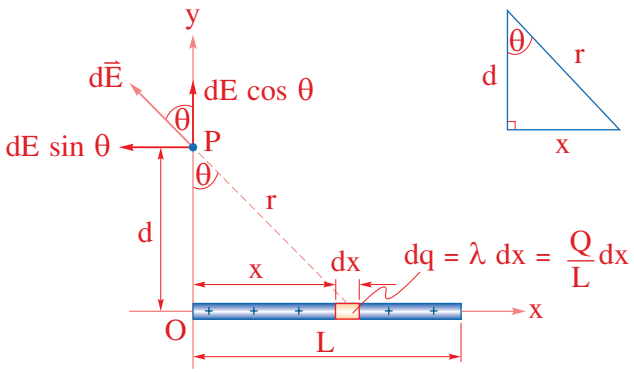


(a)

Solution

Since charge +Q is continuously and uniformly distributed on the wire, the electric field at point P must be determined by the calculus concept. That is, start by choosing an infinitesimal element of the wire, then find the electric field at point P due to the charge on that element, and finally determine the net electric field at point P by calculating the integral of the electric field from the element over the entire length of the wire. However, in this case, the electric field has no symmetry, so we must sum both the horizontal and vertical components of the electric field due to the elements, as the following detail:

Consider an infinitesimal element of length dx, far away from the origin O by distance x, as shown in Fig. (b). Charge dq on this element is obtained by $dq = \lambda\, dx$, where linear charge density $\lambda = \frac{Q}{L}$, therefore, $dq = \frac{Q}{L}\, dx$.



(b)

Charge dq is positive, so it produces electric field $d\vec{E}$ outward from the charge, making an angle θ with the positive y-axis, as shown in Fig. (b).

Charge dq and point P are separated by distance r, therefore, the magnitude of the electric field $d\vec{E}$ is

$$dE = k_0 \frac{dq}{r^2}$$

$$= k_0 \frac{\left(\frac{Q}{L} dx\right)}{r^2}$$

$$dE = \frac{k_0 Q}{L} \frac{dx}{r^2} \qquad \ldots (1)$$

In Fig. (b), when resolving electric field $d\vec{E}$ into its horizontal and vertical components, we find that the horizontal component has a magnitude of dE sin θ directed along the negative x-axis, and the vertical component has a magnitude of dE cos θ along the positive y-axis.

The magnitudes of the horizontal and vertical components of electric field $d\vec{E}$ can be found by using the expression for magnitude dE in Eq. (1) and the values of sin θ and cos θ of angle θ on the right triangle in Fig. (b):

$$dE_x = dE \sin\theta$$

$$= \frac{k_0 Q}{L} \frac{dx}{r^2} \times \frac{x}{r}$$

$$= \frac{k_0 Q}{L} \frac{x\,dx}{r^3}$$

Since $r = (x^2 + d^2)^{\frac{1}{2}}$,

$$dE_x = \frac{k_0 Q}{L} \frac{x\,dx}{(x^2 + d^2)^{\frac{3}{2}}} \quad \text{... (2)}$$

$$dE_y = dE \cos\theta$$

$$= \frac{k_0 Q}{L} \frac{dx}{r^2} \times \frac{d}{r}$$

$$= \frac{k_0 Q d}{L} \frac{dx}{r^3}$$

$$dE_y = \frac{k_0 Q d}{L} \frac{dx}{(x^2 + d^2)^{\frac{3}{2}}} \quad \text{... (3)}$$

Because the electric field due to each element has no symmetry, both the horizontal and vertical components cannot cancel to zero. Thus, the net electric field at point P has both horizontal and vertical components, which are found by integrating dE_x and dE_y in Eqs. (2) and (3) with respect to coordinate x from x = 0 to x = L, as follows:

$$E_x = \int dE_x$$

$$= \int_0^L \frac{k_0 Q}{L} \frac{x\,dx}{(x^2 + d^2)^{\frac{3}{2}}}$$

$$= \frac{k_0 Q}{L} \int_0^L \frac{x\,dx}{(x^2 + d^2)^{\frac{3}{2}}}$$

$$= \frac{k_0 Q}{2L} \int_0^L \frac{2x\,dx}{(x^2 + d^2)^{\frac{3}{2}}}$$

$$= \frac{k_0 Q}{2L}\left(\frac{-2}{(x^2 + d^2)^{\frac{1}{2}}}\right)\Bigg|_0^L$$

$$E_x = \frac{k_0 Q}{L}\left(\frac{1}{d} - \frac{1}{(L^2 + d^2)^{\frac{1}{2}}}\right)$$

$$E_y = \int dE_y$$

$$= \int_0^L \frac{k_0 Q d}{L}\frac{dx}{(x^2 + d^2)^{\frac{3}{2}}}$$

$$= \frac{k_0 Q d}{L}\int_0^L \frac{dx}{(x^2 + d^2)^{\frac{3}{2}}}$$

From $\int_0^L \frac{dx}{(x^2 + d^2)^{\frac{3}{2}}} = \frac{x}{d^2\sqrt{x^2 + d^2}}$, we get

$$E_y = \frac{k_0 Q d}{L}\left(\frac{x}{d^2\sqrt{x^2 + d^2)}}\right)\Bigg|_0^L$$

$$= \frac{k_0 Q d}{L}\left(\frac{L}{d^2\sqrt{L^2 + d^2)}} - 0\right)$$

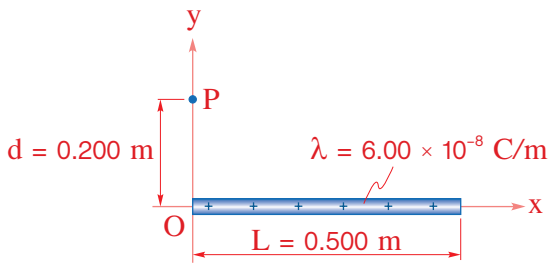
$$E_y = \frac{k_0 Q}{d\sqrt{L^2 + d^2}}$$

It can be concluded that the net electric field at point P has a horizontal component of magnitude $\frac{k_0 Q}{L}\left(\frac{1}{d} - \frac{1}{\sqrt{L^2 + d^2}}\right)$ in the negative x-direction and a vertical component of magnitude $\frac{k_0 Q}{d\sqrt{L^2 + d^2}}$ in the positive y-direction, or written in vector form as

$$\vec{E} = -\frac{k_0 Q}{L}\left(\frac{1}{d} - \frac{1}{\sqrt{L^2 + d^2}}\right)\vec{i} + \frac{k_0 Q}{d\sqrt{L^2 + d^2}}\vec{j}$$ Ans

Problem 17.72

A straight wire of length 0.500 m lies on the x-axis and has a uniformly distributed charge with a linear charge density of 6.00×10^{-8} C/m. Find the electric field at point P on the y-axis, away from the origin O by a distance of 0.200 m, as shown in the figure.



Solution

This problem requires readers to apply the electric field expression at point P, which we have already found in the past problem.

In this case, the wire has length L = 0.500 m, point P is separated from the wire's end at the origin O by distance d = 0.200 m, and the charge on the wire is $Q = \lambda L = 6.00 \times 10^{-8} \times 0.500 = 3.00 \times 10^{-8}$ C. Substituting L, d, and Q into the expression for the electric field leads to

$$E_x = \frac{k_0 Q}{L}\left(\frac{1}{d} - \frac{1}{\sqrt{L^2 + d^2}}\right)$$

$$= \frac{8.99 \times 10^9 \times 3.00 \times 10^{-8}}{0.500}\left(\frac{1}{0.200} - \frac{1}{\sqrt{0.500^2 + 0.200^2}}\right)$$

$$E_x = 1.70 \times 10^3 \text{ N/C}$$

$$E_y = \frac{k_0 Q}{d\sqrt{L^2 + d^2}}$$

$$= \frac{8.99 \times 10^9 \times 3.00 \times 10^{-8}}{0.200\sqrt{0.500^2 + 0.200^2}}$$

$$E_y = 2.50 \times 10^3 \text{ N/C}$$

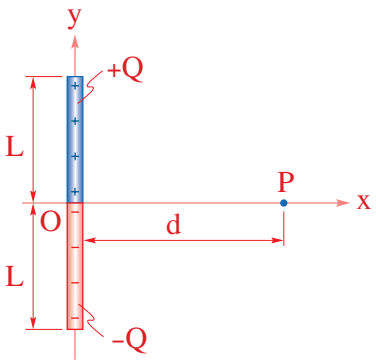
Therefore,

$$\vec{E} = -E_x \vec{i} + E_y \vec{j}$$

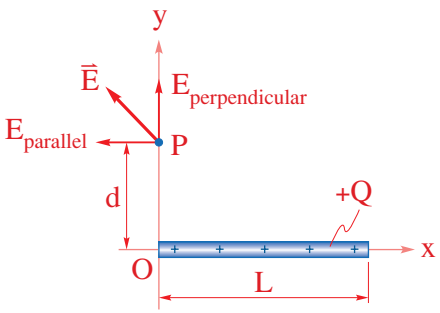
$$\vec{E} = -1.70 \times 10^3 \vec{i} + 2.50 \times 10^3 \vec{j} \text{ N/C} \quad \text{Ans}$$

Problem 17.73

Two straight wires lie on the y-axis; one end of each wire is at the origin O, as shown in Fig. (a). The upper wire has uniformly distributed charge +Q; the lower wire has uniformly distributed charge -Q. Find the net electric field at point P on the positive x-axis away from the origin O by distance d.



(a)



(b)

Solution

At point P, the net electric field is the vector sum of the electric fields due to charges +Q and -Q on the two wires. The electric field from the charge on each wire can be found by using the expressions of Prob. 17.71, as follows:

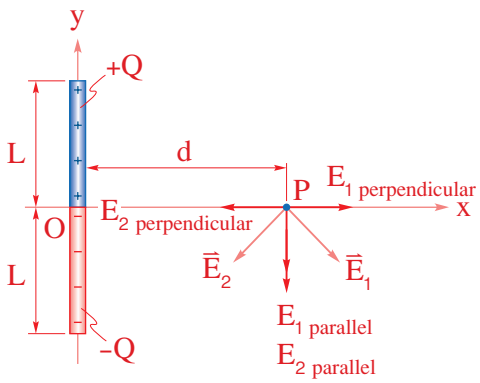
From Prob. 17.71, the net electric field at point P in Fig. (b) can be written in the components parallel and perpendicular to the wire as

$$\vec{E} = -E_{parallel} \vec{i} + E_{perpendicular} \vec{j} \quad \ldots (1)$$

where $$E_{parallel} = \frac{k_0 Q}{L}\left(\frac{1}{d} - \frac{1}{\sqrt{L^2 + d^2}}\right)$$

$$E_{\text{perpendicular}} = \frac{k_0 Q}{d\sqrt{L^2 + d^2}}$$

By applying Eq. (1) to the two wires in Fig. (a), the electric fields at point P due to the charge on each wire are shown in Fig. (c).



(c)

Figure (c) shows that the upper wire carrying charge +Q produces electric field $\vec{E}_1$ at point P directed outward from the wire. This field can be resolved into the parallel and perpendicular components as $E_{1\text{ parallel}}$ and $E_{1\text{ perpendicular}}$ along the negative y-axis and positive x-axis, respectively.

Similarly, the lower wire of charge -Q produces electric field $\vec{E}_2$ at point P directed towards the wire. When resolving this field into the parallel and perpendicular components, we find that they consist of electric fields of magnitudes $E_{2\text{ parallel}}$ and $E_{2\text{ perpendicular}}$ along the negative y-axis and negative x-axis, respectively.

Since both wires have the same length L, the same charge magnitude Q, and the end of each wire far from point P by the same distance d, then the parallel and perpendicular components of the electric fields due to each wire are equal in magnitude, that is,

$$E_{1\text{ parallel}} = E_{2\text{ parallel}} = \frac{k_0 Q}{L}\left(\frac{1}{d} - \frac{1}{\sqrt{L^2 + d^2}}\right)$$

$$E_{1\text{ perpendicular}} = E_{2\text{ perpendicular}} = \frac{k_0 Q}{d\sqrt{L^2 + d^2}}$$

At point P, the net electric field is the vector sum of the parallel and perpendicular components of the electric fields in Fig. (c). On the perpendicular direction, there are two components of magnitudes $E_{1\ \text{perpendicular}}$ and $E_{2\ \text{perpendicular}}$ in the opposite direction. Since $E_{1\ \text{perpendicular}} = E_{2\ \text{perpendicular}}$, the sum of the perpendicular components is equal to zero.

For the parallel components, there are two components of magnitudes $E_{1\ \text{parallel}}$ and $E_{2\ \text{parallel}}$ along the same negative y-axis. So the sum of these two components is the net electric field at point P, as follows:

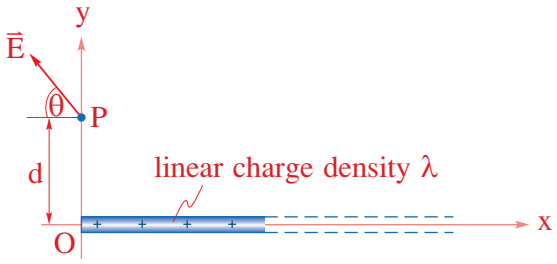
$$\vec{E}_P = (-E_{1\ \text{parallel}}\,\vec{j}\,) + (-E_{2\ \text{parallel}}\,\vec{j}\,)$$

$$= -2E_{1\ \text{parallel}}\,\vec{j}\,, \quad E_{1\ \text{parallel}} = E_{2\ \text{parallel}}$$

$$\vec{E}_P = -\frac{2k_0Q}{L}\left(\frac{1}{d} - \frac{1}{\sqrt{L^2 + d^2}}\right)\vec{j} \qquad \text{Ans}$$

Problem 17.74

An infinite straight wire lies on the x-axis, of which one end is at the origin O, as shown in Fig. (a). The wire has a continuously and uniformly distributed charge with linear charge density λ. Find the electric field's direction at point P on the y-axis, away from the end of the wire at the origin O by distance d.



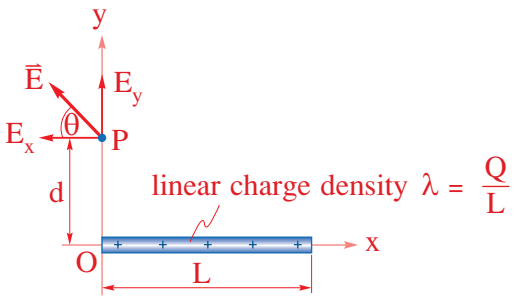
(a)

Solution

From Prob. 17.71, as shown in Fig. (b), the electric field at point P can be resolved into the x- and y-components with magnitudes of

$$E_x = \frac{k_0 Q}{L}\left(\frac{1}{d} - \frac{1}{\sqrt{L^2 + d^2}}\right)$$

$$E_y = \frac{k_0 Q}{d\sqrt{L^2 + d^2}}$$



(b)

The above expressions for E_x and E_y can be transformed into linear charge density λ by using $\lambda = \frac{Q}{L}$ and $Q = \lambda L$, as follows:

$$E_x = k_0\left(\frac{Q}{L}\right)\left(\frac{1}{d} - \frac{1}{\sqrt{L^2 + d^2}}\right)$$

$$E_x = k_0\lambda\left(\frac{1}{d} - \frac{1}{\sqrt{L^2 + d^2}}\right)$$

and

$$E_y = \frac{k_0(Q)}{d\sqrt{L^2 + d^2}}$$

$$E_y = \frac{k_0\lambda L}{d\sqrt{L^2 + d^2}}$$

From Fig. (b), the direction of electric field $\vec{E}$ at point P can be determined by angle θ, which is

$$\tan\theta = \frac{E_y}{E_x}$$

$$= \frac{\cancel{k_0}\cancel{\lambda}L}{d\sqrt{L^2+d^2}} \Big/ \cancel{k_0}\cancel{\lambda}\left(\frac{1}{d} - \frac{1}{\sqrt{L^2+d^2}}\right)$$

$$= \frac{L}{d\sqrt{L^2+d^2}\left(\frac{1}{d} - \frac{1}{\sqrt{L^2+d^2}}\right)}$$

$$= \frac{L}{\sqrt{L^2+d^2} - d}$$

$$= \frac{1}{\frac{\sqrt{L^2+d^2} - d}{L}}$$

$$\tan\theta = \frac{1}{\sqrt{1+\left(\frac{d}{L}\right)^2} - \frac{d}{L}} \qquad \ldots (1)$$

When the length of the wire is infinite, it means that L >> d, which makes $\frac{d}{L}$ approaching zero. So the value of tan θ in Eq. (1) can be calculated, as follows:

$$\tan\theta = \frac{1}{\sqrt{1+0^2} - 0} = 1$$

$$\theta = \tan^{-1}(1) = 45^\circ$$

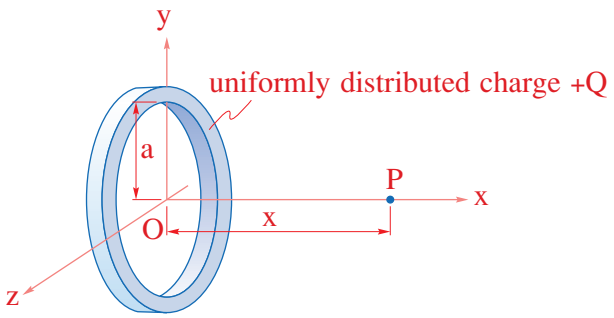
It can be concluded that the electric field at point P makes an angle of 45° above the negative x-axis, independent of separation d and linear charge density λ. Ans

Problem 17.75

A ring-shaped conductor of radius a has a uniformly distributed charge +Q along its circumference, as shown in Fig. (a).

1) Find the electric field at point P on the axis perpendicular to the ring's plane, far away from the center of the ring by distance x.

2) Find distance x that maximizes the electric field at point P.



(a)

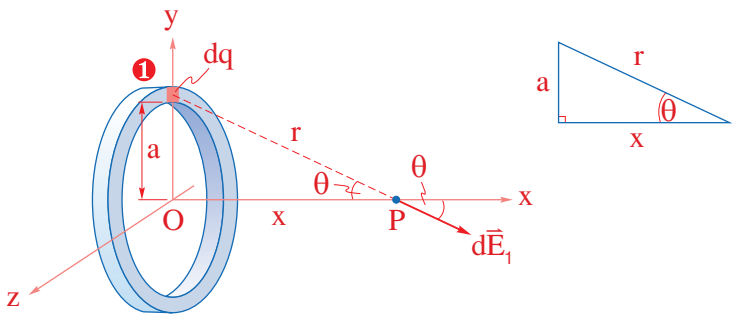
Solution

1) Begin by dividing charge +Q distributed continuously and uniformly on the ring into large amounts of infinitesimal elements. Then, find the electric field at point P due to an element. Finally, sum the electric fields due to all the elements by integrating the electric field from the element over the entire length of the ring. In this problem, the symmetry of the electric field is also taken into account, as follows:

Choose an infinitesimal element of the ring at point ❶ with positive charge dq, as shown in Fig. (b). From the figure, electric field $d\vec{E}_1$ due to charge dq is directed outward from the charge, at an angle θ below the positive x-axis.

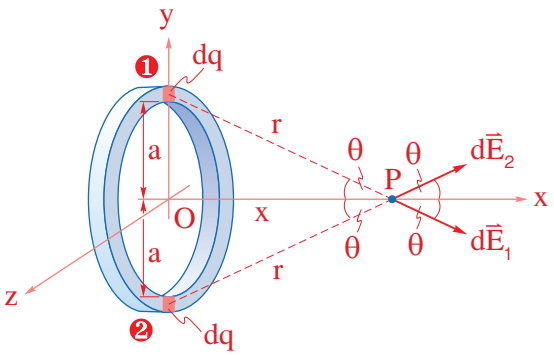
Since charge dq and point P are separated by distance r, the magnitude of the electric field $d\vec{E}_1$ is

$$dE_1 = k_0 \frac{dq}{r^2}$$



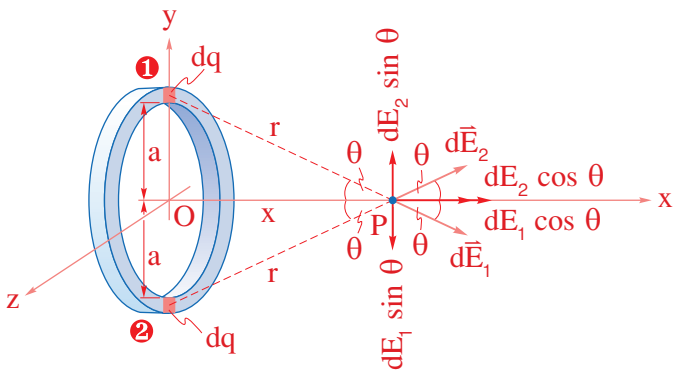
(b)

Point P is on the central axis perpendicular to the plane of the ring. Thus, at point ❷ on the opposite side to point ❶, there is another positive charge dq, as shown in Fig. (c). Positive charge dq at point ❷ is far away from point P by the same distance r as charge dq at point ❶. So charge dq at point ❷ produces electric field $d\vec{E}_2$ at point P with the same magnitude as $d\vec{E}_1$ ($dE_1 = dE_2$) and directed outward from the charge, but making the angle θ above the positive x-axis, not below, as shown in Fig. (c).



(c)

Electric fields $d\vec{E}_1$ and $d\vec{E}_2$ can be resolved into the x- and y-components, as shown in Fig. (d). Since $dE_1 = dE_2$, the y-components consist of two vectors of equal magnitudes ($dE_1 \sin\theta = dE_2 \sin\theta$) but opposite directions, making their sum equal to zero. Thus, the net electric field at point P is only the vector sum of the x-components.



(d)

At point P, the x-component of the electric field is given by

$$dE_x = dE\cos\theta = k_0\frac{dq}{r^2}\cos\theta$$

From the right triangle of angle θ in Fig. (b), we have $\cos\theta = \frac{x}{r}$ and $r = (x^2 + a^2)^{\frac{1}{2}}$, therefore,

$$dE_x = k_0\frac{dq}{r^2}\left(\frac{x}{r}\right)$$

$$= k_0\frac{x}{r^3}dq$$

$$dE_x = k_0\frac{x}{(x^2 + a^2)^{\frac{3}{2}}}dq \quad \ldots (1)$$

Because of the symmetry of the electric field at point P as discussed, the net electric field is directed along the positive x-axis. Moreover, the magnitude of the net electric field is the sum of dE_x in Eq. (1) over all the ring elements, which is given by integrating dE_x in Eq. (1), as follows:

$$E = E_x$$

$$= \int dE_x$$

$$= \int k_0\frac{x}{(x^2 + a^2)^{\frac{3}{2}}}dq$$

$$E = k_0 \frac{x}{(x^2 + a^2)^{\frac{3}{2}}} \int dq \ , \ k_0 \frac{x}{(x^2 + a^2)^{\frac{3}{2}}} \text{ is constant}$$

Because $\int dq$ equals the total charge of the ring, $\int dq = Q$, we get

$$E = \frac{k_0 x}{(x^2 + a^2)^{\frac{3}{2}}} Q$$

In conclusion, the net electric field at point P has a magnitude of $\frac{k_0 x}{(x^2 + a^2)^{\frac{3}{2}}} Q$ directed along the positive x-axis, or written in vector form as

$$\vec{E} = \frac{k_0 x}{(x^2 + a^2)^{\frac{3}{2}}} Q \, \vec{i}$$ Ans

2) Distance x that maximizes the net electric field at point P can be obtained by calculating $\frac{dE}{dx}$, then set it equal to zero, as follows:

$$\frac{dE}{dx} = \frac{d}{dx}\left(\frac{k_0 x}{(x^2 + a^2)^{\frac{3}{2}}} Q \right)$$

$$= k_0 Q \left(\frac{1(x^2 + a^2)^{\frac{3}{2}} - x\left(\frac{3}{2}\right)(x^2 + a^2)^{\frac{1}{2}}(2x)}{(x^2 + a^2)^3} \right)$$

$$= k_0 Q (x^2 + a^2)^{\frac{1}{2}} \left(\frac{x^2 + a^2 - 3x^2}{(x^2 + a^2)^3} \right)$$

$$= k_0 Q \frac{a^2 - 2x^2}{(x^2 + a^2)^{\frac{5}{2}}}$$

$$= 0$$

Therefore,

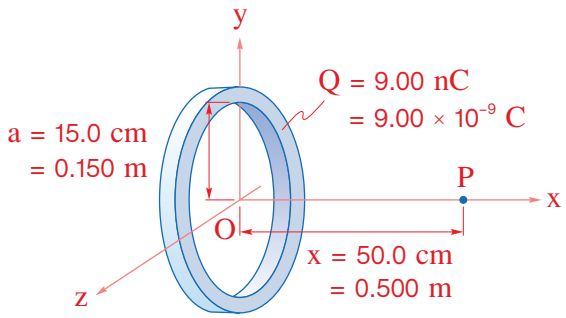
$$a^2 - 2x^2 = 0$$

$$x^2 = \frac{a^2}{2}$$

$$x = \frac{a}{\sqrt{2}} = 0.707a \qquad \text{Ans}$$

Problem 17.76

A charge of 9.00 nC is distributed uniformly on a ring of radius 15.0 cm, as shown in the figure. Find the electric field at point P on the axis perpendicular to the ring's plane, far away from the center of the ring by a distance of 50.0 cm.



Solution

This problem requires readers to apply the expression for the electric field at point P in the previous problem.

In this case, the ring has radius a = 15.0 cm = 0.150 m, point P and the ring are separated by distance x = 50.0 cm = 0.500 m, and charge Q = 9.00 nC = 9.00×10^{-9} C. So the magnitude of the electric field at point P is given by

$$E = \frac{k_0 x}{(x^2 + a^2)^{\frac{3}{2}}} Q$$

$$= \frac{8.99 \times 10^9 \times 0.500}{(0.500^2 + 0.150^2)^{\frac{3}{2}}} \times 9.00 \times 10^{-9}$$

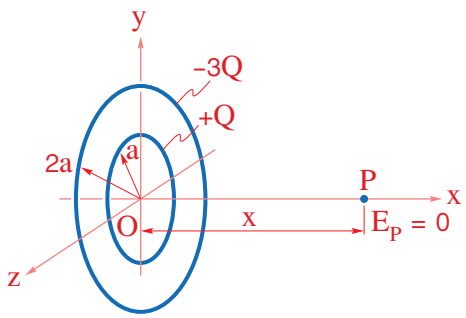
$$E = 284 \text{ N/C}$$

The electric field at point P is directed along the positive x-axis, so

$$\vec{E} = 284\,\vec{i} \text{ N/C} \qquad \text{Ans}$$

Problem 17.77

Two concentric circular rings lie on the y–z plane with a common center at the origin O, as shown in Fig. (a). The small ring has radius a and uniformly distributed charge +Q; the large ring has radius 2a and uniformly distributed charge –3Q. Find distance x of point P on the positive x–axis that has a net electric field equal to zero.

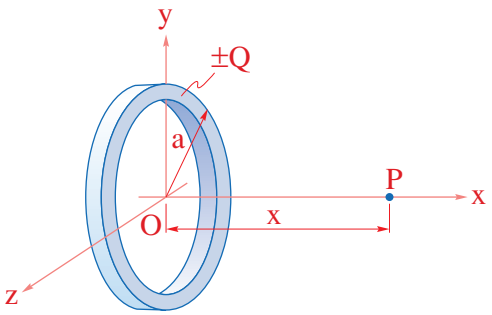


(a)

Solution

From Fig. (b) taken from Prob. 17.75, the electric field due to charge ±Q on a circular ring of radius a at point P on the x–axis, far away from the origin O by distance x, has magnitude

$$E = \frac{k_0 x}{(x^2 + a^2)^{\frac{3}{2}}} Q$$



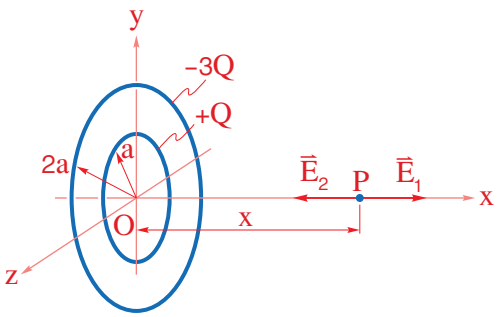
(b)

In the case of charge +Q, the electric field at point P is directed away from the ring along the positive x–axis. On the other hand, if the ring's charge is –Q, the

electric field is directed towards the ring along the negative x-axis.

When applying the above data of the electric field at point P to find the electric field in Fig. (a), we find that the ring of radius a and charge +Q produces electric field $\vec{E}_1$ directed outward from the ring along the positive x-axis, as shown in Fig. (c). Magnitude E_1 is

$$E_1 = \frac{k_0 x}{(x^2 + a^2)^{\frac{3}{2}}} Q \qquad \ldots (1)$$



(c)

Similarly, the ring of radius 2a and charge -3Q produces electric field $\vec{E}_2$ directed towards the ring along the negative x-axis, which is the opposite direction to electric field $\vec{E}_1$. Magnitude E_2 is

$$E_2 = \frac{k_0 x}{\left(x^2 + (2a)^2\right)^{\frac{3}{2}}} \times 3Q$$

$$E_2 = \frac{3k_0 x}{(x^2 + 4a^2)^{\frac{3}{2}}} Q \qquad \ldots (2)$$

From Fig. (c), it can be seen that the net electric field at point P is zero if electric fields $\vec{E}_1$ and $\vec{E}_2$ have the same magnitude. Therefore, we can determine distance x that makes the net electric field at point P equal to zero by setting E_1 equal to E_2, as follows:

$$E_1 = E_2$$

$$\frac{k_0 x}{(x^2 + a^2)^{\frac{3}{2}}} Q = \frac{3k_0 x}{(x^2 + 4a^2)^{\frac{3}{2}}} Q$$

$$\left(\frac{x^2 + 4a^2}{x^2 + a^2}\right)^{\frac{3}{2}} = 3$$

$$\frac{x^2 + 4a^2}{x^2 + a^2} = 3^{\frac{2}{3}}$$

$$x^2 + 4a^2 = 3^{\frac{2}{3}}x^2 + 3^{\frac{2}{3}}a^2$$

$$x^2(1 - 3^{\frac{2}{3}}) = (3^{\frac{2}{3}} - 4)a^2$$

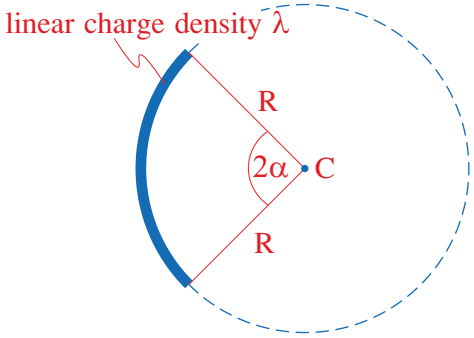
$$x = \sqrt{\frac{3^{\frac{2}{3}} - 4}{1 - 3^{\frac{2}{3}}}}\, a$$

$$x = 1.33a$$

In conclusion, when point P is at distance 1.33a from the center of the coil, the net electric field is zero. Ans

Problem 17.78

A wire is bent into a circular arc of radius R, subtending angle 2α at center C, as shown in Fig. (a). On the arc, there is a uniformly distributed charge with linear charge density λ. Find the electric field at center C due to the charge on the arc.

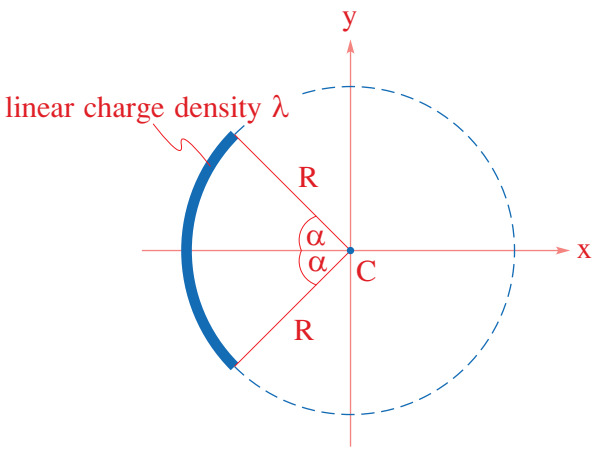


(a)

Solution

Since the charge on the wire is distributed continuously along the arc length, the electric field at center C must be obtained by the calculus method. Start by dividing the wire into infinitesimal elements, then find the electric field at center C due to the charge on an element, and finally sum all the electric fields due to every element by integration method. However, because the charge distribution is symmetrical, the symmetry of the electric field is also taken into account, as follows:

Here, we use the rectangular coordinate system x–y with the origin O at center C, as shown in Fig. (b). From the figure, the x–axis divides the subtending angle 2α into two equal angles α, making the charge distribution symmetrical to the x–axis.



(b)

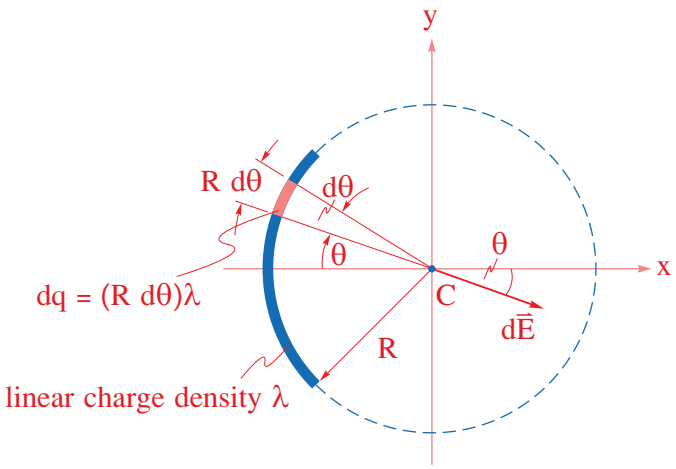
Pick up an infinitesimal element at angle θ relative to the negative x–axis and subtending angle $d\theta$ at the center, as shown in Fig. (c). The length of this element is given by the definition of an angle in radians:

$$d\theta \text{ (rad)} = \frac{\text{element's length}}{R}$$

$$\text{element's length} = R\, d\theta$$

When the length of the element is known, charge dq on the element is obtained by multiplying the element's length and the linear charge density:

$$dq = (R\, d\theta)\lambda$$



(c)

From Fig. (c), charge dq produces electric field $d\vec{E}$ at center C directed outward from the charge at angle θ below the positive x-axis. The magnitude of electric field $d\vec{E}$ is given by

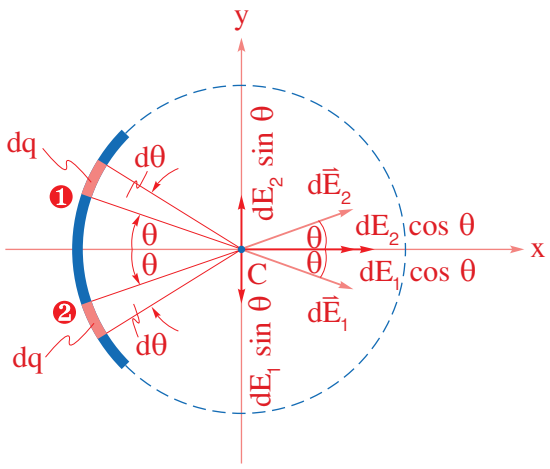
$$dE = k_0 \frac{dq}{R^2}$$

$$= k_0 \frac{R d\theta \lambda}{R^2}$$

$$dE = \frac{k_0 \lambda}{R} d\theta \quad \ldots (1)$$

Since the charge distribution is symmetrical to the x-axis, we will consider the symmetry of the electric field.

In Fig. (d), charge dq at point ❶ produces electric field $d\vec{E}_1$ at point C, at angle θ below the positive x-axis. At the same time, at point ❷, there is another charge dq that produces electric field $d\vec{E}_2$ at point C directed outward from the charge at angle θ above the positive x-axis, as shown in Fig. (d).



(d)

Since charges dq at points ❶ and ❷ are separated from center C by the same distance R, electric fields $d\vec{E}_1$ and $d\vec{E}_2$ have the same magnitude equal to the magnitude dE in Eq. (1). When resolving these two fields into the horizontal and vertical components, we find that the vertical components consist of two electric fields with magnitudes $dE_1 \sin\theta$ and $dE_2 \sin\theta$ in the opposite directions. Since $dE_1 = dE_2$, the sum of the vertical components is zero. Thus, the net electric field at point C has only the horizontal components or the x-components.

From the symmetry of the electric field as discussed, the net electric field at point C is obtained by the sum of the x-components, as follows:

$$dE_x = dE \cos\theta \qquad \text{... (2)}$$

Substitute dE from Eq. (1) into Eq. (2):

$$dE_x = \frac{k_0\lambda}{R} \cos\theta \, d\theta \qquad \text{... (3)}$$

The net electric field at point C is given by integrating dE_x in Eq. (3) from $\theta = -\alpha$ to $\theta = \alpha$:

$$E = \int dE_x$$

$$= \int_{-\alpha}^{\alpha} \frac{k_0\lambda}{R} \cos\theta \, d\theta$$

$$= \frac{k_0\lambda}{R}\sin\theta\Big|_{-\alpha}^{\alpha}$$

$$= \frac{k_0\lambda}{R}\left(\sin\alpha - \sin(-\alpha)\right)$$

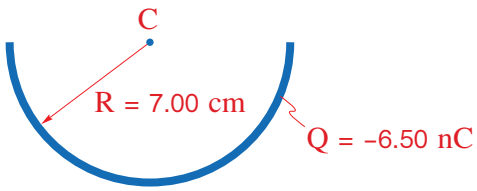
$$E = \frac{2k_0\lambda}{R}\sin\alpha, \quad \sin(-\alpha) = -\sin\alpha$$

It can be concluded that the net electric field at center C has a magnitude of $\frac{2k_0\lambda}{R}\sin\alpha$ directed along the positive x-axis, or written in vector form as

$$\vec{E} = \frac{2k_0\lambda}{R}\sin\alpha\,\vec{i}$$ Ans

Problem 17.79

A wire is bent into a semicircle of radius 7.00 cm, as shown in Fig. (a). Charge -6.50 nC is distributed uniformly along the wire. Find the resultant electric field at center C of the semicircle.



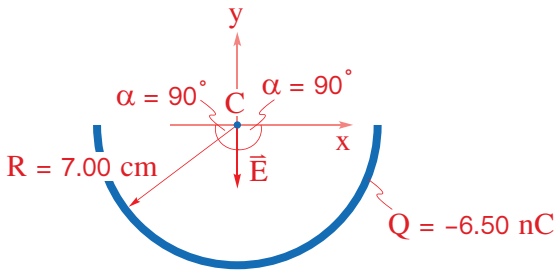
(a)

Solution

This problem requires readers to apply the expression for the electric field at the center of a circular arc in the past problem. The conclusion from that problem is when the charge on the arc is positive, the resultant electric field at the arc's center is directed outward from the arc along the axis that divides its charge into two equal parts. On the other hand, if the charge is negative, the resultant electric field's direction is altered to point towards the arc along the same axis as in the case of the positive charge. The magnitude of the resultant electric field can be found, as follows:

$$E = \frac{2k_0|\lambda|}{R}\sin\alpha \qquad \text{... (1)}$$

Set the rectangular coordinate system x–y on the semicircular wire, as shown in Fig. (b). From the figure, the y-axis is the symmetrical axis that divides the charge on the wire into two equal parts. Since the charge on the wire is negative, the resultant electric field at center C is directed towards the semicircle along the negative y-axis.



(b)

The magnitude of the resultant electric field can be found by using Eq. (1). In this case, R = 7.00 cm = 0.0700 m, $2\alpha = 180^\circ$ so $\alpha = 90^\circ$, and linear charge density $\lambda = \frac{Q}{L}$, where Q = –6.50 nC = -6.50×10^{-9} C and L is the semicircle's length equal to πR. Substitute all the data into Eq. (1) to get magnitude E:

$$E = \frac{2k_0|\lambda|}{R}\sin\alpha$$

$$= \frac{2k_0\left|\frac{Q}{\pi R}\right|}{R}\sin\alpha$$

$$= \frac{2k_0|Q|}{\pi R^2}\sin\alpha$$

$$= \frac{2 \times 8.99 \times 10^9 \times \left|-6.50 \times 10^{-9}\right|}{\pi \times 0.0700^2}\sin 90^\circ$$

$$= 7.59 \times 10^3 \text{ N/C}$$

$$E = 7.59 \text{ kN/C}$$

From the above result, we conclude that the resultant electric field at the center of the semicircle has a magnitude of 7.59 kN/C directed along the negative y-axis,

or written in vector form as

$$\vec{E} = -7.59\vec{j} \text{ kN/C}$$ Ans

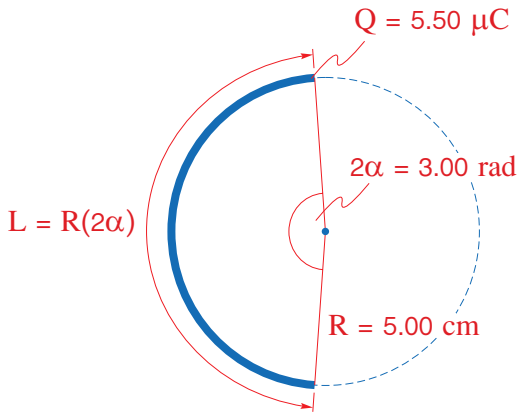
Problem 17.80

A wire is bent into a circular arc of radius 5.00 cm, subtending an angle of 3.00 rad at the center of curvature. If charge 5.50 μC is distributed uniformly over the wire's length, find the magnitude of the electric field at the center of curvature.

Solution

The magnitude of the electric field at the center of curvature due to the charge on the wire is given by the expression of Prob. 17.78, as follows:

$$E = \frac{2k_0\lambda}{R}\sin\alpha \qquad \text{... (1)}$$



Here, the problem gives radius R = 5.00 cm = 0.0500 m, angle 2α = 3.00 rad so α = 1.50 rad, and linear charge density $\lambda = \frac{Q}{L}$, where Q = 5.50 μC = 5.50 × 10^{-6} C and L = arc length subtending angle 2α = R(2α). Therefore,

$$\lambda = \frac{Q}{L} = \frac{Q}{R(2\alpha)}$$

Substitute the above quantities into Eq. (1):

$$E = \frac{2k_0\left(\frac{Q}{R2\alpha}\right)}{R}\sin\alpha$$

$$= \frac{k_0 Q}{R^2\alpha}\sin\alpha$$

$$= \frac{8.99 \times 10^9 \times 5.50 \times 10^{-6}}{0.0500^2 \times 1.50}\sin(1.50 \text{ rad})$$

$$E = 1.32 \times 10^7 \text{ N/C}$$ Ans

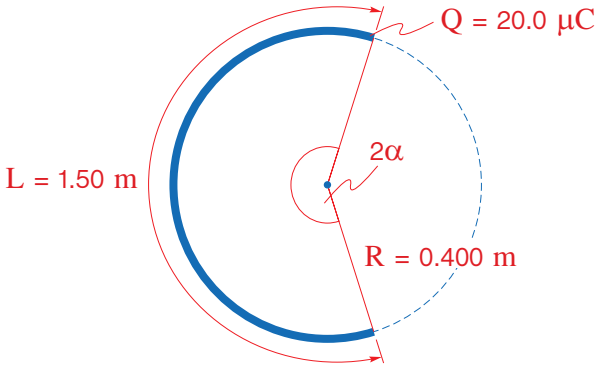
Problem 17.81

A wire of length 1.50 m is bent into a circular arc of radius 0.400 m. If charge 20.0 μC is distributed uniformly over the wire's length, find the magnitude of the electric field at the center of curvature.

Solution

Just as Prob. 17.80, the magnitude of the electric field at the center of curvature is given by

$$E = \frac{2k_0\lambda}{R}\sin\alpha \quad \text{... (1)}$$



From the figure, the problem gives radius R = 0.400 m, length L = 1.50 m, and linear charge density $\lambda = \frac{Q}{L}$, where charge Q = 20.0 μC = 20.0 × 10^{-6} C.

Angle 2α subtending the arc can be found by the definition of an angle in radians:

$$2\alpha \text{ (rad)} = \frac{L}{R}$$

$$\alpha \text{ (rad)} = \frac{L}{2R}$$

Substitute all the quantities into Eq. (1) to get magnitude E:

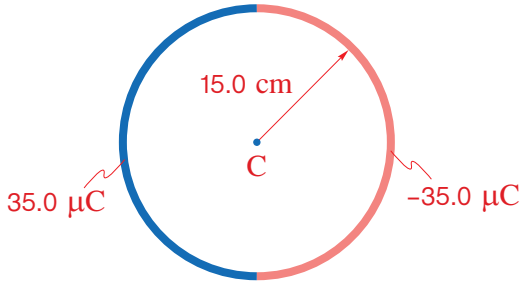
$$E = \frac{2k_0\left(\frac{Q}{L}\right)}{R}\sin\left(\frac{L}{2R}\text{ rad}\right)$$

$$= \frac{2 \times 8.99 \times 10^9 \times 20.0 \times 10^{-6}}{0.400 \times 1.50}\sin\left(\frac{1.50}{2 \times 0.400}\text{ rad}\right)$$

$$E = 5.72 \times 10^5 \text{ N/C}$$ Ans

Problem 17.82

Two semicircular wires are connected as a circle of radius 15.0 cm, as shown in Fig. (a). The left wire has a uniformly distributed charge of 35.0 μC; the right wire has a uniformly distributed charge of −35.0 μC. Find the electric field at the center of the circle.



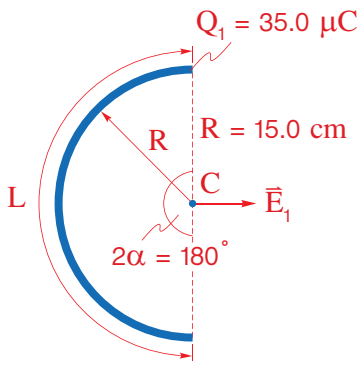
(a)

Solution

The net electric field at center C is the vector sum of the electric field due to the charges on the two semicircular wires. So start by finding the electric field due

to each semicircular wire, as follows:

Figure (b) shows the left semicircular wire with uniformly distributed charge $Q_1 = 35.0\ \mu C$. Since charge Q_1 is positive, it produces electric field $\vec{E}_1$ at center C directed outward from the charge to the right, as shown in the figure.



(b)

The magnitude of electric field $\vec{E}_1$ is given by the expression of Prob. 17.78. In this case, angle $2\alpha = 180^\circ$ so $\alpha = 90^\circ$, we get

$$E_1 = \frac{2k_0\lambda_1}{R}\sin\alpha$$

$$= \frac{2k_0\lambda_1}{R}\sin 90^\circ$$

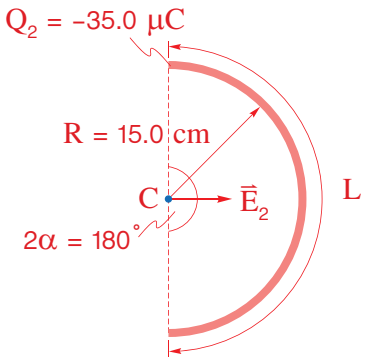
$$E_1 = \frac{2k_0\lambda_1}{R} \qquad \ldots (1)$$

We can find linear charge density λ_1 by using $\lambda_1 = \frac{Q_1}{L}$, where L is the length of the semicircle equal to πR, so $\lambda_1 = \frac{Q_1}{\pi R}$. Substituting the value of λ_1 into Eq. (1) leads to

$$E_1 = \frac{2k_0\left(\frac{Q_1}{\pi R}\right)}{R}$$

$$E_1 = \frac{2k_0Q_1}{\pi R^2} \qquad \ldots (2)$$

Figure (c) shows the right semicircular wire with uniformly distributed charge $Q_2 = -35.0\ \mu C$. Since charge Q_2 is negative, this charge creates electric field $\vec{E}_2$ at center C directed towards the wire to the right. Notice that electric fields $\vec{E}_1$ and $\vec{E}_2$ have the same direction to the right.



(c)

The magnitude of electric field $\vec{E}_2$ can be obtained by using the same steps as magnitude E_1, as follows:

$$E_2 = \frac{2k_0|\lambda_2|}{R}\sin\alpha$$

$$= \frac{2k_0\left|\frac{Q_2}{\pi R}\right|}{R}\sin 90^\circ$$

$$E_2 = \frac{2k_0|Q_2|}{\pi R^2}$$

Since $Q_1 = |Q_2| = 35.0\ \mu C$, magnitudes E_1 and E_2 in Eqs. (2) and (3) are equal, making electric fields $\vec{E}_1$ and $\vec{E}_2$ equal in magnitude and having the same direction. So the net electric field at center C, which is the vector sum of $\vec{E}_1$ and $\vec{E}_2$, is directed to the right and has a magnitude equal to the algebraic sum of E_1 and E_2, as follows:

$$E = E_1 + E_2$$

$$= \frac{2k_0Q_1}{\pi R^2} + \frac{2k_0|Q_2|}{\pi R^2}$$

$$E = 2\left(\frac{2k_0Q_1}{\pi R^2}\right),\ Q_1 = |Q_2|$$

$$= \frac{4k_0Q_1}{\pi R^2}$$

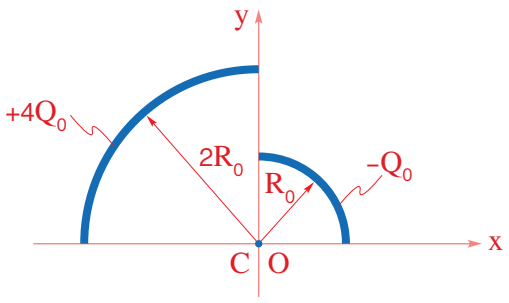
$$= \frac{4 \times 8.99 \times 10^9 \times 35.0 \times 10^{-6}}{\pi \times 0.150^2}$$

$$E = 1.78 \times 10^7\ \text{N/C}$$

In conclusion, the net electric field at the center of the circle is 1.78×10^7 N/C directed to the right. Ans

Problem 17.83

Two wires are bent into two circular arcs and positioned as in Fig. (a). The two curved wires have center C at the origin O of the rectangular coordinate system x–y. The first curved wire has radius R_0 and uniformly distributed charge $-Q_0$; the second has radius $2R_0$ and uniformly distributed charge $+4Q_0$. Find the net electric field at center C.



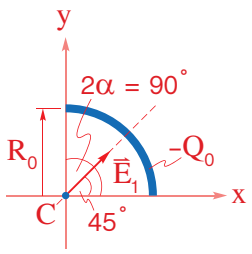
(a)

Solution

The net electric field at center C is the vector sum of the electric fields due to the charges on the two curved wires. So, we start by finding the electric field due to the charge on each wire, as follows:

Figure (b) shows electric field $\vec{E}_1$ at center C due to charge $-Q_0$ on the first curved wire. In this case, the line that divides charge $-Q_0$ into two equal and

symmetrical parts is that making a 45° angle above the positive x-axis. Since charge $-Q_0$ is negative, the electric field $\vec{E}_1$ is directed towards the wire along this line making a 45° angle above the positive x-axis, as shown in Fig. (b).



(b)

The magnitude of electric field $\vec{E}_1$ can be found by using the expression of Prob. 17.78, as follows:

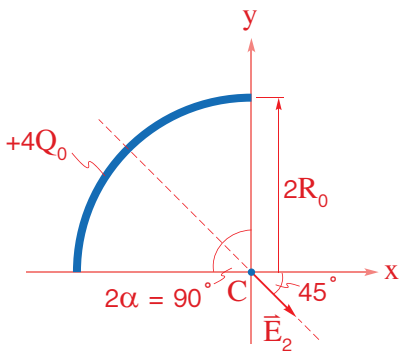
$$E_1 = \frac{2k_0|\lambda_1|}{R_1}\sin\alpha \qquad \ldots (1)$$

From Fig. (b), radius $R_1 = R_0$, angle $2\alpha = 90^\circ$ so $\alpha = 45^\circ$, and linear charge density $\lambda_1 = \frac{Q}{L} = \frac{-Q_0}{\frac{\pi R_0}{2}} = \frac{-2Q_0}{\pi R_0}$. Substitute these quantities into Eq. (1) to get

$$E_1 = \frac{2k_0\left|\frac{-2Q_0}{\pi R_0}\right|}{R_0}\sin 45^\circ$$

$$E_1 = \frac{4k_0Q_0}{\pi R_0^2}\sin 45^\circ \qquad \ldots (2)$$

Likewise, charge $+4Q_0$ on the second curved wire produces electric field $\vec{E}_2$, as shown in Fig. (c). From the figure, the line dividing charge $+4Q_0$ into two equal and symmetrical parts is that making a 45° angle below the positive x-axis. Because charge $+4Q_0$ is positive, electric field $\vec{E}_2$ is directed outward from the wire, making a 45° angle below the positive x-axis.



(c)

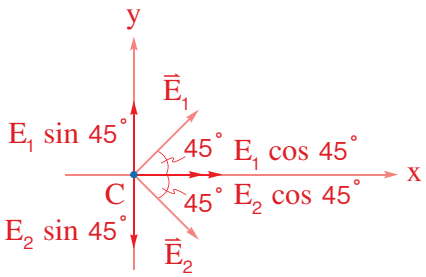
The magnitude of electric field $\vec{E}_2$ is given by the same expression as that for magnitude E_1. In this case, the wire has radius $R_2 = 2R_0$, angle $2\alpha = 90^\circ$ or $\alpha = 45^\circ$, and linear charge density $\lambda_2 = \dfrac{4Q_0}{\dfrac{\pi \times 2R_0}{2}} = \dfrac{4Q_0}{\pi R_0}$. Therefore,

$$E_2 = \frac{2k_0\lambda_2}{R_2}\sin\alpha$$

$$= \frac{2k_0\left(\dfrac{4Q_0}{\pi R_0}\right)}{2R_0}\sin 45^\circ$$

$$E_2 = \frac{4k_0Q_0}{\pi R_0^2}\sin 45^\circ \qquad \text{... (3)}$$

Notice that electric fields $\vec{E}_1$ and $\vec{E}_2$ have equal magnitude.



(d)

The net electric field at center C is the vector sum of electric fields $\vec{E}_1$ and $\vec{E}_2$, as shown in Fig. (d). In the figure, when resolving electric fields $\vec{E}_1$ and $\vec{E}_2$ into

the horizontal and vertical components, we find that the vertical components consist of two vectors of equal magnitude ($E_1 \sin 45^\circ$ and $E_2 \sin 45^\circ$, $E_1 = E_2$) but opposite directions. Therefore, the sum of the vertical components is zero, and the net electric field at point C is only the sum of the horizontal components or the x-components.

Since the x-components consist of two vectors of magnitudes $E_1 \cos 45^\circ$ and $E_2 \cos 45^\circ$ in the same positive x-direction, the net electric field also has the positive x-direction, and its magnitude is

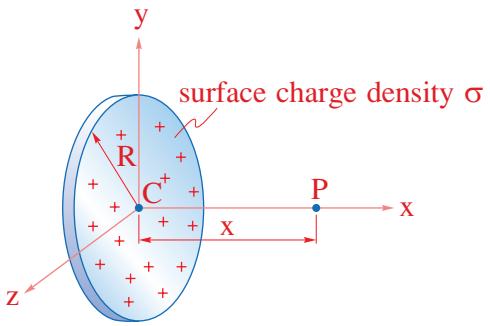
$$\begin{aligned} E &= E_1 \cos 45^\circ + E_2 \cos 45^\circ \\ &= 2E_1 \cos 45^\circ , \quad E_1 = E_2 \\ &= 2\left(\frac{4k_0Q_0}{\pi R_0^2} \sin 45^\circ\right) \cos 45^\circ , \ E_1 \text{ from Eq. (2)} \\ E &= \frac{4k_0Q_0}{\pi R_0^2} \end{aligned}$$

It can be concluded that the net electric field at the center has a magnitude of $\frac{4k_0Q_0}{\pi R_0^2}$ directed along the positive x-axis, or written in vector form as

$$\vec{E} = \frac{4k_0Q_0}{\pi R_0^2} \vec{i} \qquad \text{Ans}$$

Problem 17.84

A circular disk of radius R has a uniformly distributed charge on its surface with surface charge density σ, as shown in Fig. (a). Find the electric field at point P on the axis passing through the disk's center and perpendicular to the disk's plane, far away from the center of the disk by distance x.



(a)

Solution

A circular disk can be considered as a set of concentric rings. Thus, the electric field due to the charge on the disk is the vector sum of the electric fields due to the charges on the set of the rings forming the disk. Since in Prob. 17.75, we have found the expression for the electric field of a ring, here we will apply it to find the electric field due to the disk.

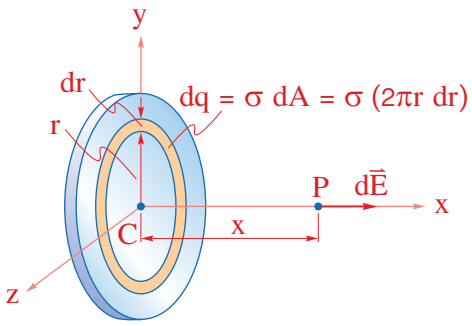
From Prob. 17.75, the symmetry of the charge on the ring causes the electric field at point P directed along the positive x-axis and having magnitude

$$E_x = \frac{k_0 x}{(x^2 + a^2)^{\frac{3}{2}}} Q \qquad \ldots (1)$$

According to the concept that a disk is a set of concentric rings, consider a ring of radius r and thickness dr, as shown in Fig. (b). This ring has a surface area equal to 2πr dr. So the charge on the ring is equal to the multiple of the surface charge density and the surface area of the ring, that is, dq = (2πr dr) σ. Electric field $d\vec{E}$ due to charge dq on the ring is directed along the positive x-axis, and its magnitude is given by substituting Q = dq = (2πr dr) σ and a = r into Eq. (1), as follows:

$$dE = k_0 \frac{x}{(x^2 + r^2)^{\frac{3}{2}}} (2\pi r\, dr)\sigma$$

$$dE = \frac{k_0 x}{(x^2 + r^2)^{\frac{3}{2}}} (2\pi r\, \sigma) dr \qquad \ldots (2)$$



(b)

At point P, the net electric field due to the total charge on the disk is the vector sum of the electric fields from all the rings forming the disk. Since the electric field due to the ring points along the positive x-axis, the net electric field is also directed along the positive x-axis. The magnitude of the net electric field is obtained by integrating dE in Eq. (2) with respect to r from r = 0 to r = R, as follows:

$$E = \int dE$$

$$= \int_0^R \frac{k_0 x}{(x^2 + r^2)^{\frac{3}{2}}} (2\pi r\, \sigma) dr$$

$$= k_0 x \pi\sigma \int_0^R \frac{2r\, dr}{(x^2 + r^2)^{\frac{3}{2}}}, \; k_0 x\pi\sigma \text{ is constant}$$

$$= k_0 x \pi\sigma \int_0^R (x^2 + r^2)^{-\frac{3}{2}} d(x^2 + r^2)$$

$$= k_0 x \pi\sigma \left(\frac{(x^2 + r^2)^{-\frac{1}{2}}}{-\frac{1}{2}} \right) \Bigg|_0^R$$

$$E = 2\pi k_0 \sigma \left(1 - \frac{x}{(x^2 + R^2)^{\frac{1}{2}}} \right)$$

It can be concluded that the net electric field at point P has a magnitude of $2\pi k_0 \sigma \left(1 - \frac{x}{(x^2 + R^2)^{\frac{1}{2}}} \right)$ directed along the positive x-axis, or written in vector form as

$$\vec{E} = 2\pi k_0 \sigma \left(1 - \frac{x}{(x^2 + R^2)^{\frac{1}{2}}} \right) \vec{i}$$ Ans

Problem 17.85

From Prob. 17.84, find the electric field at point P in the case of an infinite disk, which means radius R of the disk is much greater than distance x (R >> x).

Solution

According to Prob. 17.84, for a disk of radius R and uniformly distributed charge with surface charge density σ, the electric field at point P away from the disk by distance x is directed along the positive x-axis and has magnitude

$$E = 2\pi k_0 \sigma \left(1 - \frac{x}{(x^2 + R^2)^{\frac{1}{2}}} \right) \quad \ldots (1)$$

In the case of an infinite disk, radius R of the disk is much larger than distance x (R >> x). Thus, we can estimate the quantity in Eq. (1), as follows:

$$\frac{x}{(x^2 + R^2)^{\frac{1}{2}}} \simeq \frac{x}{(R^2)^{\frac{1}{2}}}, \quad x^2 + R^2 \simeq R^2$$

$$\simeq \frac{x}{R}$$

$$\simeq 0 \ , \ R >> x$$

From the above result, we can approximate the magnitude of the electric field at point P as

$$E \simeq 2\pi k_0 \sigma(1 - 0)$$

$$\simeq 2\pi k_0 \sigma$$

$$E \simeq \frac{\sigma}{2\varepsilon_0} \ , \ 2\pi k_0 = \frac{1}{2\varepsilon_0}$$

The expression for magnitude E shows that, at any point not on the surface, the electric field due to the infinite disk has a constant magnitude equal to $\frac{\sigma}{2\varepsilon_0}$. It can also be applied to any shaped sheet because $E = \frac{\sigma}{2\varepsilon_0}$ does not depend on radius R.

We can conclude that

"For an infinite sheet of any shape with surface charge density σ, the magnitude of the electric field at upper or lower points of the sheet is $E = \frac{\sigma}{2\varepsilon_0}$."

Remarks

As discussed, the infinite sheet means the sheet with a large size compared to the separation from point P to the sheet. In real situations, an infinite sheet can be considered as a finite sheet to which point P is very close compared to the distance from point P to the edge of the sheet. In such case, the electric field has a constant magnitude equal to $E = \frac{\sigma}{2\varepsilon_0}$ and is directed away from the sheet if the charge on the sheet is positive, or towards the sheet if negative.

Problem 17.86

From Prob. 17.84, find the electric field at point P when point P is very far from the disk.

Solution

Just as Prob. 17.85, the electric field at point P is directed along the positive x-axis and has the following magnitude:

$$E = 2\pi k_0 \sigma \left(1 - \frac{x}{(x^2 + R^2)^{\frac{1}{2}}} \right) \quad \text{... (1)}$$

When point P is very far from the disk, distance x from point P to the disk is much greater than radius R of the disk ($x >> R$). Therefore, we can estimate the magnitude of the electric field in Eq. (1), as follows:

Transform Eq. (1):

$$E = 2\pi k_0 \sigma \left(1 - \frac{1}{\frac{1}{x}\sqrt{x^2 + R^2}} \right)$$

$$E = 2\pi k_0 \sigma \left(1 - \frac{1}{\sqrt{1 + \frac{R^2}{x^2}}} \right) \quad \text{... (2)}$$

Since $x >> R$, making $\frac{R^2}{x^2} << 1$. So the quantity $\sqrt{1 + \frac{R^2}{x^2}}$ in Eq. (2) can be approximated with the binomial distribution, as follows:

When $\beta << 1$, we can approximate that $(1 + \beta)^n \simeq 1 + n\beta$. Apply this approximated value to $\sqrt{1 + \frac{R^2}{x^2}}$ or $\left(1 + \frac{R^2}{x^2}\right)^{\frac{1}{2}}$, where $\frac{R}{x} << 1$, as follows:

$$(1+\beta)^n \simeq 1 + n\beta,\ \beta << 1$$

$$\left(1+\frac{R^2}{x^2}\right)^{\frac{1}{2}} = (1+\beta)^{\frac{1}{2}},\quad \beta = \frac{R^2}{x^2} << 1$$

$$\simeq 1 + \frac{1}{2}\beta$$

$$\left(1+\frac{R^2}{x^2}\right)^{\frac{1}{2}} \simeq 1 + \frac{1}{2}\frac{R^2}{x^2}$$

Substitute the above result into Eq. (2):

$$E \simeq 2\pi k_0\sigma\left(1 - \frac{1}{1+\frac{1}{2}\frac{R^2}{x^2}}\right)$$

$$\simeq 2\pi k_0\sigma\left(\frac{1+\frac{1}{2}\frac{R^2}{x^2} - 1}{1+\frac{1}{2}\frac{R^2}{x^2}}\right)$$

$$\simeq 2\pi k_0\sigma\left(\frac{\frac{1}{2}\frac{R^2}{x^2}}{\frac{2x^2+R^2}{2x^2}}\right)$$

$$E \simeq 2\pi k_0\sigma\left(\frac{R^2}{2x^2+R^2}\right) \quad \ldots (3)$$

Assume that the total charge on the disk is equal to Q, so $\sigma = \frac{Q}{\pi R^2}$. Substituting $\sigma = \frac{Q}{\pi R^2}$ into Eq. (3) leads to

$$E \simeq 2\pi k_0\left(\frac{Q}{\pi R^2}\right)\left(\frac{R^2}{2x^2+R^2}\right)$$

$$E \simeq 2k_0Q\left(\frac{1}{2x^2+R^2}\right) \quad \ldots (4)$$

Since $x >> R$, $2x^2 + R^2 \simeq 2x^2$. Therefore, we can estimate magnitude E of the electric field in Eq. (4), as follows:

$$E \simeq 2k_0Q\left(\frac{1}{2x^2}\right)$$

$$E \simeq \frac{k_0Q}{x^2} \qquad \text{Ans}$$

In conclusion, when point P is very far from the disk, the electric field at point P is the same as that of the total charge Q on the disk condensed to a point charge Q. It makes sense because an observer at point P very far from the disk looks the disk as a point.

Problem 17.87

A disk of radius 0.200 m has a uniformly distributed charge with a surface charge density of $-6.00\ \mu C/m^2$. Find the electric field at point P on the central axis perpendicular to the disk's plane, 0.150 m from the center of the disk.

Solution

This problem requires readers to apply the expression for the electric field at point P due to the charge on the disk of Prob. 17.84. In this case, the surface charge density is negative, so the disk has a negative charge, making the electric field at point P directed towards the disk's center along the axis perpendicular to the disk's plane. Magnitude E of the electric field is

$$E = 2\pi k_0 |\sigma| \left(1 - \frac{x}{(x^2 + R^2)^{\frac{1}{2}}}\right) \qquad \ldots (1)$$

The problem gives $\sigma = -6.00\ \mu C/m^2 = -6.00 \times 10^{-6}\ C/m^2$, $R = 0.200$ m, and $x = 0.150$ m. Substitute these quantities into Eq. (1):

$$E = 2\pi \times 8.99 \times 10^{9} \times |-6.00 \times 10^{-6}| \left(1 - \frac{0.150}{(0.150^2 + 0.200^2)^{\frac{1}{2}}} \right)$$

$$E = 1.36 \times 10^{5} \text{ N/C}$$

In conclusion, the electric field at point P is 1.36×10^{5} N/C directed towards the center of the disk. Ans

Problem 17.88

A disk has radius R and a uniformly distributed charge on its surface. Find a position on the axis passing through the center and perpendicular to the disk's plane at which the electric field's magnitude is only 25% of that at the center.

Solution

The magnitude of the electric field at a point along the axis passing through the center and perpendicular to the disk's plane far away from the center of the disk by distance x, in which the disk has radius R and a uniformly distributed charge with surface charge density σ, is given by

$$E = 2\pi k_0 \sigma \left(1 - \frac{x}{(x^2 + R^2)^{\frac{1}{2}}} \right) \quad \ldots (1)$$

The problem requires position x at which the electric field's magnitude is 25% of that at the center. Given E_C and E be the magnitudes of the electric fields at the center and the position x, respectively. So we can write the condition as said in mathematical form as

$$\frac{E}{E_C} = 0.25 \quad \ldots (2)$$

The magnitude of the electric field at the center is obtained by substituting x = 0 into Eq. (1), as follows:

$$E_C = 2\pi k_0 \sigma \left(1 - \frac{0}{(0^2 + R^2)^{\frac{1}{2}}} \right)$$

$$E_C = 2\pi k_0 \sigma \qquad \text{... (3)}$$

Substitute E and E_C from Eqs. (1) and (3) into Eq. (2) to find position x:

$$\frac{E}{E_C} = 0.25$$

$$\frac{2\pi k_0 \sigma \left(1 - \frac{x}{(x^2 + R^2)^{\frac{1}{2}}} \right)}{2\pi k_0 \sigma} = 0.25$$

$$1 - \frac{x}{(x^2 + R^2)^{\frac{1}{2}}} = 0.25$$

$$\frac{x^2}{x^2 + R^2} = 0.75^2$$

$$(1 - 0.75^2)x^2 = 0.75^2 R^2$$

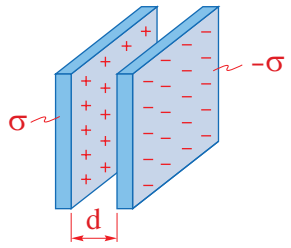
$$x = \sqrt{\frac{0.75^2}{1 - 0.75^2}}\, R$$

$$x = 1.13R$$

The above result shows that the position at which the magnitude of the electric field is 25% of that at the center is far from the center by 1.13 times the radius of the disk. Ans

Problem 17.89

Find the electric field between two large thin sheets separated by distance d, as shown in Fig. (a). Given distance d is small compared to the sheet's size and the two sheets have surface charge densities σ and −σ.



(a)

Solution

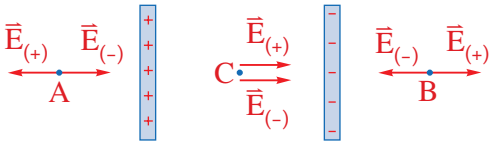
When distance d is small compared to the size of the sheets, we can treat them as infinite sheets. Thus, the electric field due to the charge on the sheets in Fig. (a) can be obtained by using the expression of Prob. 17.85. That is, if the surface charge density is positive, the electric field is directed outward from the sheet, whereas, if negative, the electric field is directed towards the sheet. The magnitude of the electric field is given by

$$E = \frac{\sigma}{2\varepsilon_0}$$

Applying the above conclusion to both the sheets, it is found that the sheet of surface charge density σ produces electric field $\vec{E}_{(+)}$ directed outward the sheet, and the sheet of surface charge density −σ creates electric field $\vec{E}_{(-)}$ directed towards the sheet. Electric fields $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ have the same magnitude, that is

$$E_{(+)} = E_{(-)} = \frac{\sigma}{2\varepsilon_0}$$

Electric fields $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ at various field points are shown in Fig. (b).



(b)

In Fig. (b), points outside the region between the sheets, either the left side at point A or the right side at point B, always have two electric fields due to both sheets in the opposite directions. Since the two sheets produce electric fields equal in magnitude, the net electric field at those points is always zero.

For the region between the sheets, such as point C in Fig. (b), electric fields $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ have the same direction, directed from the positively charged sheet to the negatively charged sheet. Therefore, the net electric field is also directed to the right and has magnitude

$$E = E_{(+)} + E_{(-)}$$

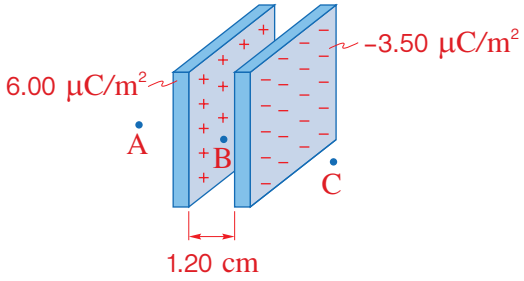
$$= \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

It can be concluded that the electric field between large sheets has a constant value equal to $\frac{\sigma}{\varepsilon_0}$, no matter how close to the sheets or at the midpoint between the sheets, but those field points must be so far away from the edges of the sheets enough to be estimated as infinite sheets. Ans

Problem 17.90

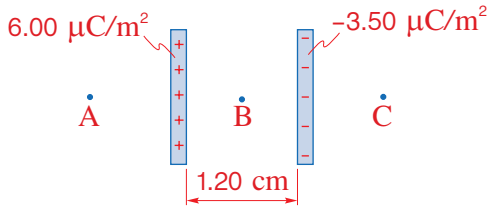
Two large flat plates are placed 1.20 cm apart, as shown in Fig. (a). Given the plates so large that we can treat them as infinite plates. If the left and the right plates have a uniformly distributed charge with surface charge densities of 6.00 $\mu C/m^2$ and −3.50 $\mu C/m^2$, respectively, find the net electric fields at points A, B, and C.



(a)

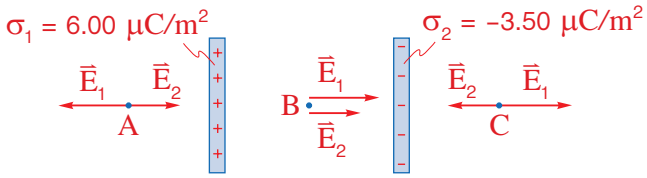
Solution

We will find the net electric field at each field point from the edge view of the plates, as shown in Fig. (b).



(b)

If the charge on the plate is positive, the electric field is directed outward from the sheet, whereas, if negative, directed towards the plate. Let the electric fields due to the left and right plates be $\vec{E}_1$ and $\vec{E}_2$, respectively. So we can represent electric fields $\vec{E}_1$ and $\vec{E}_2$ at points A, B, and C, as shown in Fig. (c).



(c)

The magnitudes of electric fields $\vec{E}_1$ and $\vec{E}_2$ are obtained by the expression of Prob. 17.85, as follows:

$$E = \frac{\sigma}{2\varepsilon_0}$$

$$E_1 = \frac{\sigma_1}{2\varepsilon_0}$$

$$E_2 = \frac{|\sigma_2|}{2\varepsilon_0}$$

Since $\sigma_1 > |\sigma_2|$, $E_1 > E_2$. It causes the arrow for electric field $\vec{E}_1$ in Fig. (c) is longer than that of electric field $\vec{E}_2$.

Since $E_1 > E_2$, at point A in Fig. (c), the net electric field is directed to the left along electric field $\vec{E}_1$ and has magnitude

$$E_A = E_1 - E_2$$

$$= \frac{\sigma_1}{2\varepsilon_0} - \frac{|\sigma_2|}{2\varepsilon_0}$$

$$= \frac{\sigma_1 - |\sigma_2|}{2\varepsilon_0}$$

$$= \frac{6.00 \times 10^{-6} - \left|-3.50 \times 10^{-6}\right|}{2 \times 8.85 \times 10^{-12}}$$

$$E_A = 1.41 \times 10^5 \text{ N/C}$$

In conclusion, the net electric field at point A is 1.41×10^5 N/C directed to the left. Ans

From Fig. (c), at point B, electric fields $\vec{E}_1$ and $\vec{E}_2$ have the same direction to the right, so the net electric field is also directed to the right and has magnitude

$$E_B = E_1 + E_2$$

$$= \frac{\sigma_1}{2\varepsilon_0} + \frac{|\sigma_2|}{2\varepsilon_0}$$

$$= \frac{\sigma_1 + |\sigma_2|}{2\varepsilon_0}$$

$$= \frac{6.00 \times 10^{-6} + \left|-3.50 \times 10^{-6}\right|}{2 \times 8.85 \times 10^{-12}}$$

$$E_B = 5.37 \times 10^5 \text{ N/C}$$

In conclusion, the net electric field at point B is 5.37×10^5 N/C directed to the right. Ans

At point C in Fig. (c), electric fields $\vec{E}_1$ and $\vec{E}_2$ are in the opposite directions. Therefore, the net electric field points to the right along electric field $\vec{E}_1$ and has magnitude

$$E_C = E_1 - E_2$$

$$= E_A$$

$$E_C = 1.41 \times 10^5 \text{ N/C}$$

In conclusion, the net electric field at point C is 1.41×10^5 N/C directed to the right. Ans

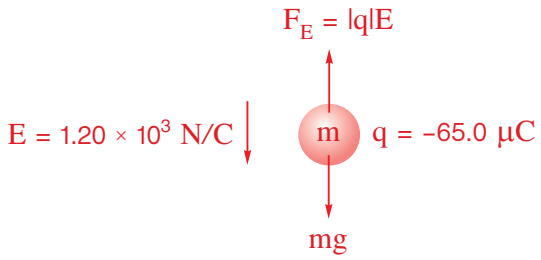
"Motion of Electric Charges in Electric Field"

Problem 17.91

An object of charge −65.0 μC is placed in a uniform electric field of magnitude 1.20×10^3 N/C in the downward direction. If the object floats at rest, find the mass of the object.

Solution

"An object floats at rest" means it is in equilibrium. So we start by drawing its free-body diagram, then use the equilibrium condition for forces to find the object's mass, as follows:



Since the object has a negative charge, the electric field produces an electric force on the object in the upward direction opposite to the electric field's direction. So the forces acting on the object consist of weight mg in the downward direction and the electric force in the upward direction, as shown in the figure. The magnitude of the electric force is $F_E = |q|E$.

Applying the equilibrium condition for the vertical forces leads to

$$\begin{aligned} mg &= F_E \\ &= |q|E \\ m &= \frac{|q|E}{g} \\ &= \frac{\left|-65.0 \times 10^{-6}\right| \times 1.20 \times 10^3}{9.80} \\ &= 7.96 \times 10^{-3}\ \text{kg} \\ m &= 7.96\ \text{g} \end{aligned}$$

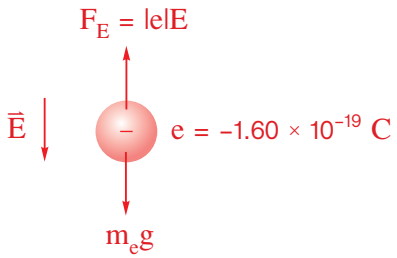
Ans

Problem 17.92

An electron floats at rest in an electric field. Find the magnitude and direction of the electric field.

Solution

"An electron floats at rest" means the electron is in equilibrium. So begin by drawing the free-body diagram of the electron, then use the equilibrium condition for forces to find the magnitude and direction of the electric field, as follows:



From the figure, the forces acting on the electron consist of weight $m_e g$ in the downward direction, where m_e = electron's mass = 9.11×10^{-31} kg, and electric force $\vec{F}_E$.

Since the weight has a downward direction, the electric force must be directed upward to make the electron in equilibrium. The electron's charge is negative; the electric force in the upward direction gives us that electric field $\vec{E}$ is in the downward direction. The magnitude of the electric field is obtained by the equilibrium condition for the vertical forces, as follows:

$$|e|E = m_e g$$

$$E = \frac{m_e g}{|e|}$$

$$= \frac{9.11 \times 10^{-31} \times 9.80}{\left|-1.60 \times 10^{-19}\right|}$$

$$E = 5.58 \times 10^{-11} \text{ N/C}$$

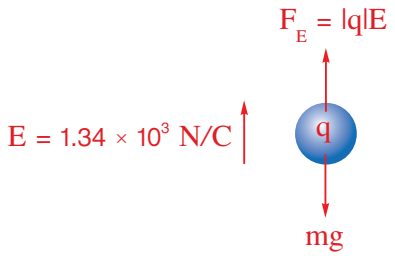
In conclusion, the electric field is 5.58×10^{-11} N/C in the downward direction. Ans

Problem 17.93

A particle has a mass of 8.15 g and floats in an electric field of magnitude 1.34×10^3 N/C in the upward direction. Find the charge of the particle.

Solution

"A particle floats at rest" means the particle is in equilibrium. So we begin by drawing the free-body diagram of the particle, then use the equilibrium condition for forces to find the particle's charge, as follows:



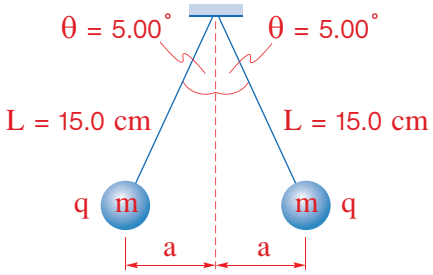
From the figure, we see that weight mg acts on the particle in the downward direction, so electric force $\vec{F}_E$ must be in the upward direction to get the resultant force on the particle equal to zero. Since electric field $\vec{E}$ and electric force $\vec{F}_E$ have the same upward direction, charge q of the particle is positive. The magnitude of charge q is given by the equilibrium condition for the vertical forces:

$$
\begin{aligned}
|q|E &= mg \\
|q| &= \frac{mg}{E} \\
&= \frac{8.15 \times 10^{-3} \times 9.80}{1.34 \times 10^3} \\
&= 59.6 \times 10^{-6}\ \text{C} \\
|q| &= 59.6\ \mu\text{C}
\end{aligned}
$$

In conclusion, the particle has a charge of +59.6 μC. Ans

Problem 17.94

Two identical small spheres have the same charge q and mass m. They are suspended in equilibrium by two insulating wires of length L = 15.0 cm and at an angle $\theta = 5.00^\circ$ with the vertical, as shown in Fig. (a). Find the magnitude of the charge on each sphere, given m = 65.0 g.



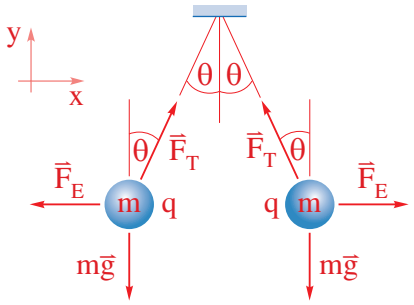
$m = 65.0\ g = 65.0 \times 10^{-3}\ kg$

(a)

Solution

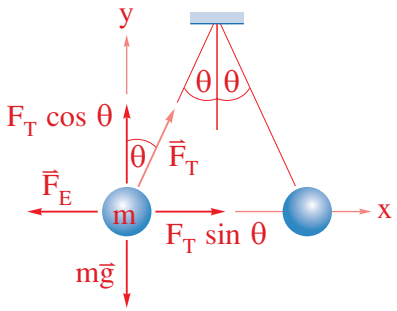
Since both spheres have the same charge q, the electric forces between them are repulsive. If, in the beginning, the spheres are hung vertically, the repulsive electric forces push them apart, then damped oscillation from air resistance occurs. Finally, the spheres are in equilibrium at the position in Fig. (a). Thus, we would find the charge on each sphere by considering their equilibrium.

Start by drawing the free-body diagram for each sphere, as in Fig. (b). From the figure, it can be seen that the force system on each sphere is symmetrical with respect to the vertical at the connecting point of the two wires so that we can analyze the equilibrium from either sphere. Here, the sphere on the left is selected.



(b)

The forces acting on the sphere consist of tension $\vec{F}_T$ from the wire that makes an angle θ with the vertical, weight $m\vec{g}$ in the downward direction, and repulsive electric force $\vec{F}_E$ directed along the negative x-axis. Because the sphere is in equilibrium, we use equations $\sum \vec{F}_x = 0$ and $\sum \vec{F}_y = 0$, as follows:



(c)

$$\sum \vec{F}_x = 0$$

$$(+F_T \sin \theta) + (-F_E) = 0$$

$$F_T \sin \theta = F_E \quad \text{... (1)}$$

$$\sum \vec{F}_y = 0$$

$$(+F_T \cos \theta) + (-mg) = 0$$

$$F_T \cos \theta = mg \quad \text{... (2)}$$

Divide Eq. (1) by Eq. (2) to find the magnitude of the electric force:

$$\frac{F_T \sin \theta}{F_T \cos \theta} = \frac{F_E}{mg}$$

$$\tan\theta = \frac{F_E}{mg}$$

$$F_E = mg\tan\theta \qquad \ldots (3)$$

Next, determine magnitude F_E in terms of charge q. In Fig. (a), the separation between the spheres is distance 2a. Since a = L sin θ, separation r = 2a = 2L sin θ and the magnitude of the electric force is

$$F_E = k_0\frac{|q_1q_2|}{r^2}$$

$$= k_0\frac{|q \times q|}{(2L\sin\theta)^2}$$

$$F_E = k_0\frac{|q^2|}{(2L\sin\theta)^2} \qquad \ldots (4)$$

Substitute the value of F_E from Eq. (4) into Eq. (3):

$$k_0\frac{|q^2|}{(2L\sin\theta)^2} = mg\tan\theta$$

$$|q^2| = \frac{4mgL^2\tan\theta\sin^2\theta}{k_0}$$

$$|q| = 2L\sin\theta\sqrt{\frac{mg\tan\theta}{k_0}} \qquad \ldots (5)$$

Substitute $m = 65.0 \times 10^{-3}$ kg, L = 0.150 m, g = 9.80 m/s^2, and $\theta = 5.00^\circ$ into Eq. (5) to get the magnitude of the charge:

$$|q| = 2 \times 0.150 \sin 5.00^\circ \sqrt{\frac{65.0 \times 10^{-3} \times 9.80 \tan 5.00^\circ}{8.99 \times 10^9}}$$

$$= 65.1 \times 10^{-9}\ \text{C}$$

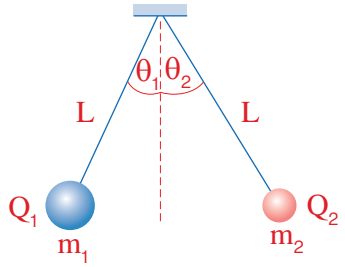
$$|q| = 65.1\ \text{nC}$$

It can be concluded that the magnitude of the charge on each sphere is 65.1 nC.

Ans

Problem 17.95

Two small spheres have masses m_1 and m_2 and are suspended by two wires of the same length L, as shown in Fig. (a). If both spheres have the same type of charge and the wires make angles θ_1 and θ_2 with the vertical, find the ratio $\frac{\theta_1}{\theta_2}$. Given the angles θ_1 and θ_2 so small.



(a)

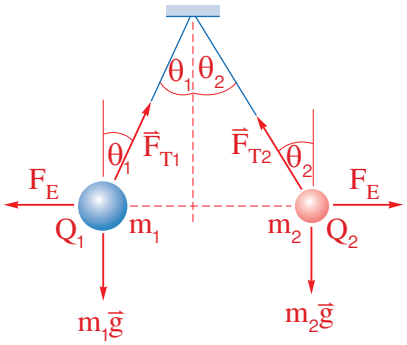
Solution

Because the charges on both spheres are of the same type, the electric forces between the spheres are repulsive, and they can be in equilibrium, as shown in Fig. (a).

In this case, angles θ_1 and θ_2 are assigned to be very small. Therefore, both spheres can be approximated to be aligned on the same horizontal level. It means the electric forces between the spheres are repulsive along the horizontal.

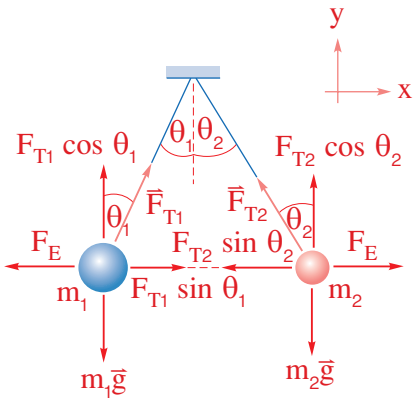
The ratio $\frac{\theta_1}{\theta_2}$ can be obtained by considering the equilibrium of each sphere. Begin by drawing the free-body diagrams of both spheres, as shown in Fig. (b).

In Fig. (b), weight $m_1\vec{g}$ and $m_2\vec{g}$ are in the downward direction, and tensions $\vec{F}_{T1}$ and $\vec{F}_{T2}$ make angles θ_1 and θ_2 with the vertical which their magnitudes are not necessarily the same. The interesting thing is the electric force between the spheres: whether the charges on both spheres are equal or not, the electric forces on the two spheres always have the same magnitude. So, in Fig. (b), the electric force on each sphere is in the direction pushing apart and has the same magnitude F_E.



(b)

Analyze the equilibrium of each sphere by equations $\sum \vec{F}_x = 0$ and $\sum \vec{F}_y = 0$, where tensions $\vec{F}_{T1}$ and $\vec{F}_{T2}$ must be resolved into the horizontal and vertical components, as in Fig. (c):



(c)

The sphere on the left:

$$\sum \vec{F}_x = 0$$

$$(+F_{T1} \sin \theta_1) + (-F_E) = 0$$

$$F_{T1} \sin \theta_1 = F_E \quad \text{... (1)}$$

$$\sum \vec{F}_y = 0$$

$$(+F_{T1} \cos \theta_1) + (-m_1 g) = 0$$

$$F_{T1} \cos \theta_1 = m_1 g \quad \text{... (2)}$$

Divide Eq. (1) by Eq. (2):

$$\frac{F_{T1}\sin\theta_1}{F_{T1}\cos\theta_1} = \frac{F_E}{m_1 g}$$

$$\tan\theta_1 = \frac{F_E}{m_1 g} \quad \text{... (3)}$$

Since angle θ_1 is small, we can estimate that $\tan\theta_1 \simeq \sin\theta_1 \simeq \theta_1$(rad). Substituting this estimated value into Eq. (3) gives

$$\theta_1 = \frac{F_E}{m_1 g} \quad \text{... (4)}$$

The sphere on the right:

$$\sum \vec{F}_x = 0$$

$$(-F_{T2}\sin\theta_2) + (+F_E) = 0$$

$$F_{T2}\sin\theta_2 = F_E \quad \text{... (5)}$$

$$\sum \vec{F}_y = 0$$

$$(+F_{T2}\cos\theta_2) + (-m_2 g) = 0$$

$$F_{T2}\cos\theta_2 = m_2 g \quad \text{... (6)}$$

Divide Eq. (5) by Eq. (6):

$$\frac{F_{T2}\sin\theta_2}{F_{T2}\cos\theta_2} = \frac{F_E}{m_2 g}$$

$$\tan\theta_2 = \frac{F_E}{m_2 g} \quad \text{... (7)}$$

Because angle θ_2, just as θ_1, is small, $\tan\theta_2 \simeq \theta_2$(rad). Substitute this approximated value into Eq. (7) to get the expression for angle θ_2:

$$\theta_2 = \frac{F_E}{m_2 g} \quad \text{... (8)}$$

Divide Eq. (4) by Eq. (8) to find the ratio $\frac{\theta_1}{\theta_2}$, as follows:

$$\frac{\theta_1}{\theta_2} = \frac{\frac{F_E}{m_1 g}}{\frac{F_E}{m_2 g}}$$

$$\frac{\theta_1}{\theta_2} = \frac{m_2}{m_1}$$ Ans

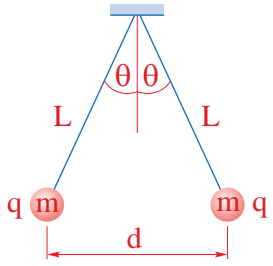
Remarks

1) The ratio $\frac{\theta_1}{\theta_2}$ depends only on the mass ratio, not on the charges of the spheres.

2) If $m_1 = m_2$, the ratio $\frac{\theta_1}{\theta_2} = 1$ and $\theta_1 = \theta_2$. It means that the spheres suspended by the wires are symmetrical to the vertical, but if the masses are not the same, angles $\theta_1 \neq \theta_2$ and the symmetry does not happen.

Problem 17.96

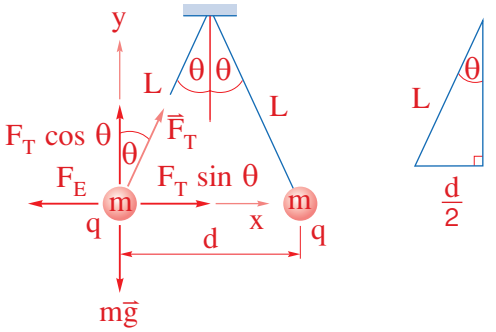
Two small spheres have equal mass m and charge q. Each sphere is suspended by a wire of length L, making a tiny angle θ with the vertical and is in equilibrium, as shown in Fig. (a). Show that $d = \left(\frac{2k_0 q^2 L}{mg}\right)^{\frac{1}{3}}$.



(a)

Solution

Two spheres of equal mass m are hung symmetrically with the vertical. Here, we would analyze the equilibrium on the left sphere to find distance d between the spheres, as follows:



(b)

Figure (b) shows the free-body diagram for the left sphere. The forces acting on the sphere consist of weight $m\vec{g}$ in the downward direction, tension $\vec{F}_T$ resolved into the horizontal and vertical components, and the electric force of magnitude F_E in the direction pushing the sphere away to the left. When using the equilibrium conditions for forces along the x- and y-axes, we get

$$\sum \vec{F}_x = 0$$

$$(+F_T \sin \theta) + (-F_E) = 0$$

$$F_T \sin \theta = F_E \quad \text{... (1)}$$

$$\sum \vec{F}_y = 0$$

$$(+F_T \cos \theta) + (-mg) = 0$$

$$F_T \cos \theta = mg \quad \text{... (2)}$$

Divide Eq. (1) by Eq. (2):

$$\frac{F_T \sin \theta}{F_T \cos \theta} = \frac{F_E}{mg}$$

$$\tan \theta = \frac{F_E}{mg} \quad \text{... (3)}$$

Since angle θ is small, we can estimate that $\tan \theta \simeq \sin \theta$. From the right triangle of angle θ in Fig. (b), $\sin \theta = \frac{\frac{d}{2}}{L} = \frac{d}{2L}$. Therefore,

$$\tan \theta \simeq \sin \theta = \frac{d}{2L} \quad \text{... (4)}$$

Magnitude F_E of the electric force is given by considering two charges q, distance d apart, as follows:

$$F_E = k_0 \frac{q \times q}{d^2}$$

$$F_E = \frac{k_0 q^2}{d^2} \qquad \text{... (5)}$$

Substitute tan θ and F_E from Eqs. (4) and (5) into Eq. (3) to find distance d:

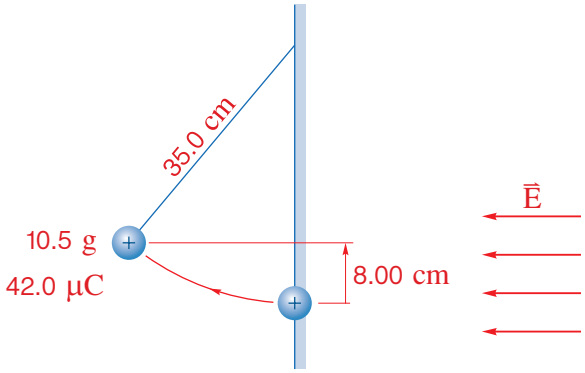
$$\frac{d}{2L} = \frac{\frac{k_0 q^2}{d^2}}{mg}$$

$$d^3 = \frac{k_0 q^2}{mg} \times 2L$$

$$d = \left(\frac{2k_0 q^2 L}{mg} \right)^{\frac{1}{3}} \qquad \text{Ans}$$

Problem 17.97

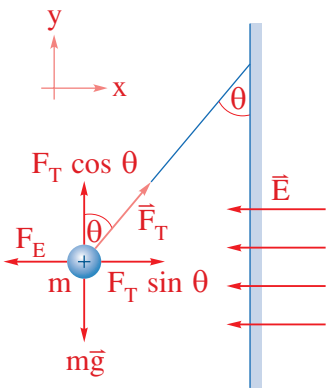
A small sphere, mass 10.5 g, is tied to one end of a thread of length 35.0 cm, which the other end is attached to the wall, as shown in Fig. (a). The sphere is in a uniform electric field directed along the horizontal. When charging the sphere to 42.0 μC, it is in equilibrium at the position above the lowest point by distance 8.00 cm. Find the electric field.



(a)

Solution

Electric field $\vec{E}$ produces an electric force on the sphere. This force and the other forces cause the sphere in equilibrium. Therefore, we will analyze the equilibrium of the sphere to find electric field $\vec{E}$.



(b)

Start by drawing the free-body diagram for the sphere, as shown in Fig. (b). From the figure, the first thing that should be identified is that tension $\vec{F}_T$ points upward to the right, so the sphere is in equilibrium only if the electric force, which is directed in the horizontal along the electric field, is directed to the left. Since the sphere has a positive charge, the electric field is in the same direction as the electric force. That means the electric field is directed to the left or along the negative x-axis.

Now, the direction of the electric field is known, then we will find the magnitude of the electric field by using the equilibrium conditions $\sum \vec{F}_x = 0$ and $\sum \vec{F}_y = 0$, where tension $\vec{F}_T$ is resolved into the horizontal and vertical components, as follows:

$$\sum \vec{F}_x = 0$$

$$(+F_T \sin \theta) + (-F_E) = 0$$

$$F_T \sin \theta = F_E \quad \text{... (1)}$$

$$\sum \vec{F}_y = 0$$

$$(+F_T \cos \theta) + (-mg) = 0$$

$$F_T \cos \theta = mg \quad \text{... (2)}$$

Divide Eq. (1) by Eq. (2) and substitute the magnitude of the electric field by $F_E = qE$:

$$\frac{F_T \sin\theta}{F_T \cos\theta} = \frac{F_E}{mg}$$

$$\tan\theta = \frac{F_E}{mg}$$

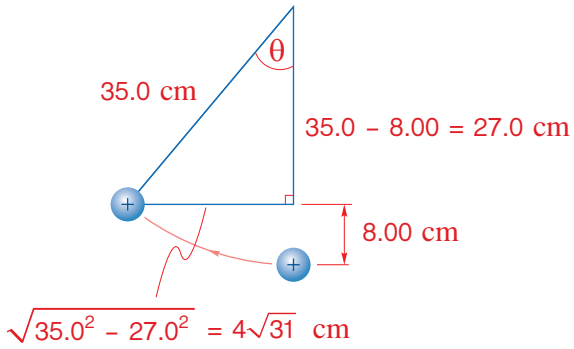
$$= \frac{qE}{mg}$$

Therefore,
$$E = \frac{mg \tan\theta}{q} \quad \text{... (3)}$$

The value of tan θ can be determined by the right triangle in Fig. (c). From the figure, $\tan\theta = \frac{4\sqrt{31}}{27.0}$. Substitute this value and other quantities into Eq. (3) to find the magnitude of the electric field:

$$E = \frac{10.5 \times 10^{-3} \times 9.80 \times \frac{4\sqrt{31}}{27.0}}{42.0 \times 10^{-6}}$$

$$E = 2.02 \times 10^3 \text{ N/C}$$



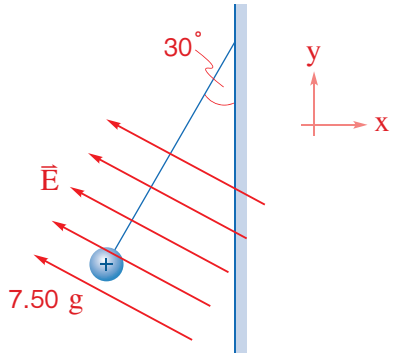
(c)

In conclusion, the electric field is 2.02×10^3 N/C directed to the left, or written in vector form as

$$\vec{E} = -2.02 \times 10^3 \, \vec{i} \text{ N/C}$$ Ans

Problem 17.98

A particle of mass 7.50 g is tied to one end of the thread, which the other end is attached to the wall, as shown in Fig. (a). The particle is in a uniform electric field $\vec{E} = -7.50 \times 10^3\,\vec{i} + 3.50 \times 10^3\,\vec{j}$ N/C and equilibrium at the position making an angle of 30° with the wall. Find the charge on the particle.



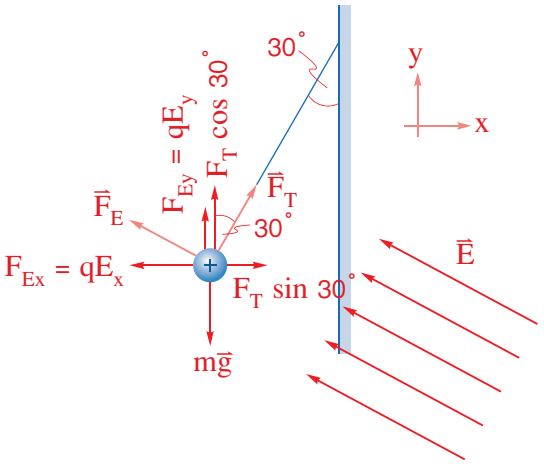
$\vec{E} = -7.50 \times 10^3\,\vec{i} + 3.50 \times 10^3\,\vec{j}$ N/C

(a)

Solution

Given the particle has charge q. Electric field $\vec{E}$ produces electric force $\vec{F}_E$ on the particle, and this force, combined with other forces, keeps the particle in equilibrium. Therefore, we would analyze the equilibrium of the particle to find charge q.

Begin by drawing the free-body diagram of the particle, as shown in Fig. (b). From the data in the figure, we can identify the type of the particle's charge. Electric field $\vec{E}$ is directed upward to the left, so electric force $\vec{F}_E$ may be directed upward to the left in the same direction of the electric field, or directed downward to the right opposite to the electric field's direction, depending on the charge's type. Since tension $\vec{F}_T$ is directed upward to the right, the electric force that makes the particle in equilibrium must be directed upward to the left. This direction is the same as the direction of the electric field. So the charge on the particle is positive.



(b)

Electric field $\vec{E} = -7.50 \times 10^3\,\vec{i} + 3.50 \times 10^3\,\vec{j}$ N/C shows that the horizontal component has magnitude $E_x = 7.50 \times 10^3$ N/C along the negative x-axis or directed to the left, and the vertical component has magnitude $E_y = 3.50 \times 10^3$ N/C along the positive y-axis or directed upward. So electric force $\vec{F}_E$ can be resolved into the horizontal and vertical components of magnitudes qE_x and qE_y in the same direction as the components of the electric field, as shown in Fig. (b).

Analyze the force equilibrium of the particle by using equations $\sum \vec{F}_x = 0$ and $\sum \vec{F}_y = 0$, where tension $\vec{F}_T$ must be resolved into the x- and y-components, as follows:

$$\sum \vec{F}_x = 0$$

$$(+F_T \sin 30^\circ) + (-qE_x) = 0$$

$$F_T \sin 30^\circ = qE_x \qquad \ldots (1)$$

$$\sum \vec{F}_y = 0$$

$$(+F_T \cos 30^\circ) + (+qE_y) + (-mg) = 0$$

$$F_T \cos 30^\circ + qE_y = mg \qquad \ldots (2)$$

Since charge q is required, let us find the value of F_T in Eq. (1), then substitute it into Eq. (2):

Eq. (1): $$F_T = \frac{qE_x}{\sin 30^\circ}$$

Eq. (2): $$\frac{qE_x}{\sin 30^\circ} \times \cos 30^\circ + qE_y = mg$$

$$q(E_x \cot 30^\circ + E_y) = mg$$

$$q = \frac{mg}{E_x \cot 30^\circ + E_y}$$

$$= \frac{7.50 \times 10^{-3} \times 9.80}{7.50 \times 10^3 \cot 30^\circ + 3.50 \times 10^3}$$

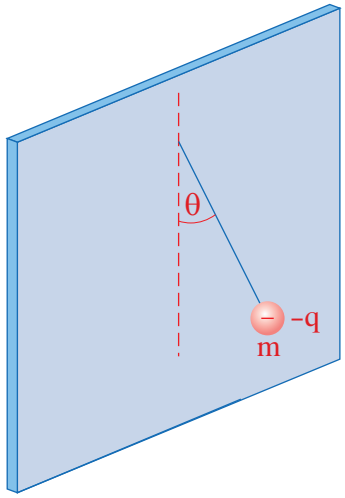
$$= 4.46 \times 10^{-6}\ \text{C}$$

$$q = 4.46\ \mu\text{C}$$

It can be concluded that the particle has a charge of +4.46 μC. Ans

Problem 17.99

A large flat plate is aligned vertically and has a uniformly distributed charge, as shown in Fig. (a). A small sphere of mass m and charge –q is tied to one end of a thread, which the other end is attached to the plate. It appears that the sphere is in equilibrium at angle θ with the plate. Show that the surface charge density of the plate is $-\frac{2mg\varepsilon_0 \tan\theta}{q}$.

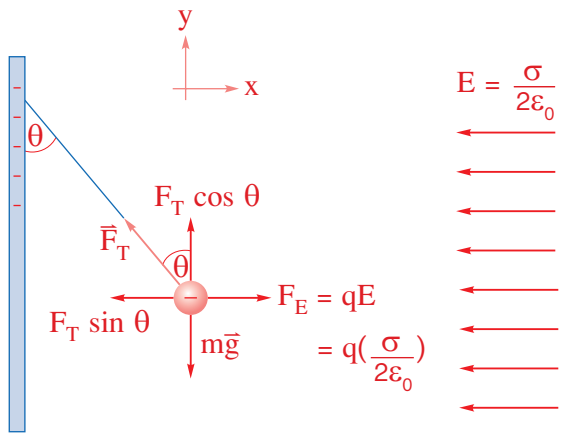


(a)

Solution

An electric field due to a large (infinite) plate is directed outward from the plate if the plate's charge is positive, and directed towards the plate if negative. The magnitude of the electric field is $E = \frac{\sigma}{2\varepsilon_0}$. Use this data to analyze the equilibrium of the sphere to find the surface charge density, as follows:

For convenience, we will consider the equilibrium of the sphere from the edge view of the plate, as shown in Fig. (b).



(b)

Figure (b) shows the free-body diagram of the sphere. From the figure, we see that tension $\vec{F}_T$ is directed upward to the left, so electric force $\vec{F}_E$, which is directed horizontally along electric field $\vec{E}$, must be directed to the right along the positive x-axis to make the sphere in equilibrium. Since the sphere has charge -q, the electric field and electric force are in the opposite direction. This lets us know that the electric field is directed to the left along the negative x-axis towards the plate; therefore, the charge on the plate is negative. Given the surface charge density equal to $-\sigma$.

The electric field due to the plate has magnitude $E = \frac{\sigma}{2\varepsilon_0}$. Therefore, the electric force on the sphere has magnitude $F_E = qE = q\left(\frac{\sigma}{2\varepsilon_0}\right)$ directed to the right, as shown in Fig. (b). Analyze the force equilibrium of the sphere to determine σ, as follows:

$$\sum \vec{F}_x = 0$$

$$(-F_T \sin \theta) + \left(+\frac{q\sigma}{2\varepsilon_0}\right) = 0$$

$$F_T \sin\theta = \frac{q\sigma}{2\varepsilon_0} \quad \text{... (1)}$$

$$\sum \vec{F}_y = 0$$

$$(+F_T \cos\theta) + (-mg) = 0$$

$$F_T \cos\theta = mg \quad \text{... (2)}$$

Divide Eq. (1) by Eq. (2):

$$\frac{F_T \sin\theta}{F_T \cos\theta} = \frac{\frac{q\sigma}{2\varepsilon_0}}{mg}$$

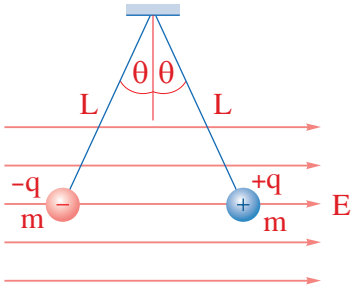
$$\tan\theta = \frac{q\sigma}{2mg\varepsilon_0}$$

$$\sigma = \frac{2mg\varepsilon_0 \tan\theta}{q}$$

Because the charge on the plate is negative, the surface charge density is $-\sigma = -\frac{2mg\varepsilon_0 \tan\theta}{q}$. Ans

Problem 17.100

Two small spheres have the same mass m. Each sphere is suspended by a thread of length L, as shown in Fig (a). The spheres have charges –q and +q and are in a uniform electric field of magnitude E directed to the right. If both spheres are in equilibrium at angle θ with the vertical, find the magnitude of the electric field.

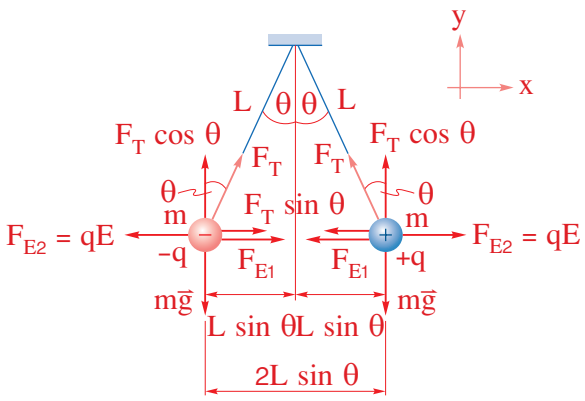


(a)

Solution

The electric field produces electric forces on both spheres. These forces, combined with other forces, make the spheres in equilibrium. Therefore, we would analyze the equilibrium of the spheres to find the magnitude of the electric field.

Begin by drawing the free-body diagrams of the spheres, as shown in Fig. (b). From the figure, the electric forces on each sphere have two forces. The first is the electric force between charges –q and +q on the spheres; this force is attractive, whose magnitude is assigned to F_{E1}. The second force is the force due to electric field $\vec{E}$, which is directed to the right. For the left sphere of charge –q, the electric force is directed to the left opposite to the electric field's direction. On the other hand, the electric force on the right sphere of charge +q has the same direction as the electric field, pointing to the right. The electric force due to the electric field has magnitude $F_{E2} = qE$.



(b)

Considering the force system of each sphere, we find that it is symmetrical to the vertical. Therefore, we can analyze the equilibrium of either sphere. Here, the sphere on the left is used.

Analyze the force equilibrium on the left sphere by using equations $\sum \vec{F}_x = 0$ and $\sum \vec{F}_y = 0$, where magnitude F_{E1} of the electric force between charges –q and +q at distance 2L sin θ apart is obtained by

$$F_{E1} = k_0 \frac{|q_1 q_2|}{r^2}$$

$$= k_0 \frac{|(+q) \times (-q)|}{(2L \sin\theta)^2}$$

$$F_{E1} = \frac{k_0 q^2}{4L^2 \sin^2\theta}$$

$$\sum \vec{F}_x = 0$$

$$(+F_{E1}) + (+F_T \sin\theta) + (-F_{E2}) = 0$$

$$\frac{k_0 q^2}{4L^2 \sin^2\theta} + F_T \sin\theta = qE \qquad \text{... (1)}$$

$$\sum \vec{F}_y = 0$$

$$(+F_T \cos\theta) + (-mg) = 0$$

$$F_T \cos\theta = mg \qquad \text{... (2)}$$

Since we want to know magnitude E of the electric field in Eq. (1), find F_T from Eq. (2), then substitute it into Eq. (1), as follows:

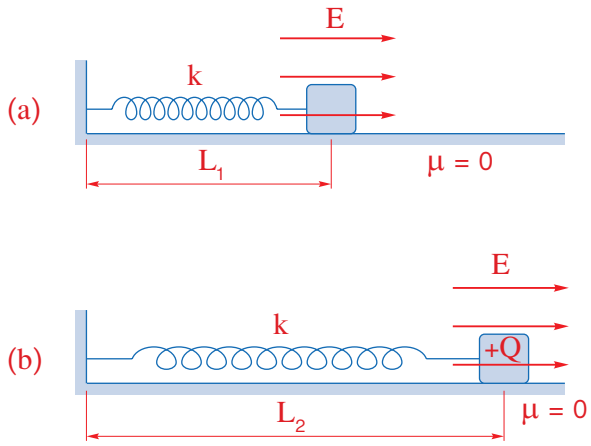
Eq. (2): $$F_T = \frac{mg}{\cos\theta}$$

Eq. (1): $$\frac{k_0 q^2}{4L^2 \sin^2\theta} + \left(\frac{mg}{\cos\theta}\right)\sin\theta = qE$$

$$E = \frac{k_0 q}{4L^2 \sin^2\theta} + \frac{mg}{q}\tan\theta \qquad \text{Ans}$$

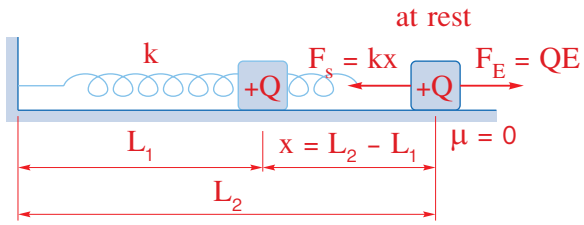
Problem 17.101

An object is placed on a frictionless horizontal floor in a region of a uniform electric field that has magnitude E directed to the right, as shown in Fig. (a). The object is attached to a spring with spring constant k, and the other end of the spring is attached to the wall. The spring is initially not stretched or compressed, and the object is far away from the wall by distance L_1. Then put charge +Q on the object, causing an electric force on the object and stretching the spring until the object is at rest at distance L_2 from the wall, as in Fig. (b). Find the charge on the object.



Solution

When putting charge +Q on the object, the electric field produces an electric force on the object that has magnitude $F_E = QE$ in the same direction as the electric field, that is, directed to the right, as shown in Fig. (c).



(c)

The electric force pulls the object to move to the right. While the object is traveling, the spring is stretched. So the object experiences another force from the

spring directed to the left. Therefore, the position of the object at rest is one that the object is in equilibrium between the electric force and the spring force. Given this position is the point that the spring is stretched by distance x, so the spring force has magnitude $F_s = kx$, where distance $x = L_2 - L_1$, as shown in Fig. (c). Use the equilibrium condition for the horizontal forces to find charge Q, as follows:

$$F_E = F_s$$

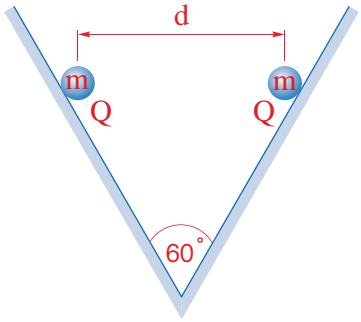
$$QE = kx$$

$$= k(L_2 - L_1)$$

Therefore, $$Q = \frac{k(L_2 - L_1)}{E}$$ Ans

Problem 17.102

Two frictionless, flat sheets are assembled as a groove at an angle of 60°, as shown in Fig. (a). Then put two small spheres with the same mass m and charge +Q in the groove. It turns out that the spheres move to their equilibrium positions when they are separated by distance d. Find charge Q on the spheres.



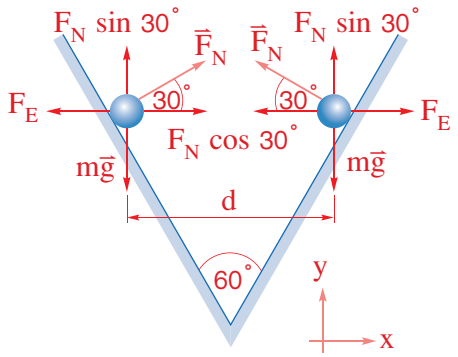
(a)

Solution

When the spheres are in equilibrium, one force acting on each sphere is the electric force due to charge +Q on the spheres. Therefore, we would analyze the equilibrium of the spheres to find charge Q.

Starting by drawing the free-body diagrams of the spheres, as shown in Fig. (b). From the figure, the forces acting on each sphere consist of weight $m\vec{g}$ directed

downward, normal force $\vec{F}_N$ from the sheet perpendicular to the sheet, and the horizontal electric force $\vec{F}_E$ in the direction pushing the spheres apart since the spheres have the same charge +Q.



(b)

Figure (b) shows that the force systems of the spheres are symmetrical, so we can analyze the equilibrium of either sphere. Here, the sphere on the left is used.

Normal force $\vec{F}_N$ is perpendicular to the inclined sheet that the sphere touches. Using the 60° angle of the groove, normal force $\vec{F}_N$ makes an angle of 30° above the horizontal. The first step is to resolve the normal force into the horizontal and vertical components, as shown in Fig. (b). Then analyze the equilibrium of the sphere by using equations $\sum \vec{F}_x = 0$ and $\sum \vec{F}_y = 0$, as follows:

$$\sum \vec{F}_x = 0$$

$$(+F_N \cos 30^\circ) + (-F_E) = 0$$

$$F_N \cos 30^\circ = F_E \quad \text{... (1)}$$

$$\sum \vec{F}_y = 0$$

$$(+F_N \sin 30^\circ) + (-mg) = 0$$

$$F_N \sin 30^\circ = mg \quad \text{... (2)}$$

Divide Eq. (2) by Eq. (1):

$$\frac{F_N \sin 30^\circ}{F_N \cos 30^\circ} = \frac{mg}{F_E}$$

$$\tan 30^\circ = \frac{mg}{F_E} \qquad \ldots (3)$$

The spheres have equal charge +Q and are separated by distance d, so the magnitude of the electric force is given by

$$F_E = k_0 \frac{|q_1 q_2|}{r^2}$$

$$= k_0 \frac{Q \times Q}{d^2}$$

$$F_E = k_0 \frac{Q^2}{d^2} \qquad \ldots (4)$$

Substitute F_E from Eq. (4) into Eq. (3) to find charge Q:

$$\tan 30^\circ = \frac{mg}{k_0 \frac{Q^2}{d^2}}$$

$$Q^2 = \frac{mgd^2}{k_0 \tan 30^\circ}$$

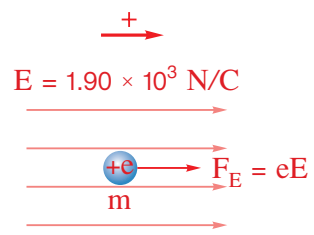
$$Q = d\left(\frac{mg}{k_0 \tan 30^\circ}\right)^{\frac{1}{2}} \qquad \text{Ans}$$

Problem 17.103

A proton is placed in a uniform electric field of magnitude 1.90×10^3 N/C. Find the acceleration of the proton due to the electric force.

Solution

A proton has mass $m = 1.67 \times 10^{-27}$ kg and charge $+e = 1.60 \times 10^{-19}$ C. When placing the proton in electric field $\vec{E}$, it experiences an electric force of magnitude $F_E = eE$ in the same direction as the electric field. The acceleration of the proton due to the electric force is given by Newton's second law of motion.



Set the electric field's direction as positive, therefore,

$$\sum \vec{F} = m\vec{a}$$

$$+eE = m\vec{a}$$

$$\vec{a} = +\frac{eE}{m}$$

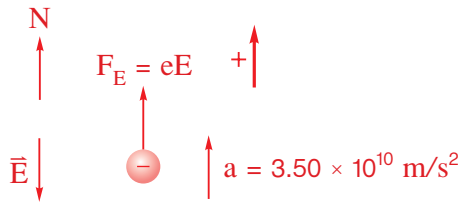
$$= +\frac{1.60 \times 10^{-19} \times 1.90 \times 10^{3}}{1.67 \times 10^{-27}}$$

$$= +1.82 \times 10^{11} \text{ m/s}^2$$

$\vec{a} = 1.82 \times 10^{11}$ m/s^2 in the same direction as the electric field Ans

Problem 17.104

An electric field causes an electron to accelerate at 3.50×10^{10} m/s^2 due to the north. Find the electric field.



Solution

An electron has mass m = 9.11×10^{-31} kg and charge −e = -1.60×10^{-19} C. So when the electron is in an electric field, it experiences an electric force that can accelerate it. As said, we would find the electric field by using Newton's second law of motion.

Electric force $\vec{F}_E$ is the resultant force causing the electron to accelerate to the north (neglecting the electron's weight compared to the electric force since its much less magnitude). Because the resultant force and the acceleration must be in the same direction, the electric force is also directed to the north. The electric force acts on the electron of a negative charge, so the electric field is directed to the south opposite to the electric force's direction, as shown in the figure.

The electric force has magnitude $F_E = eE$ and is directed to the north. Set the north direction as positive, then use Newton's second law of motion to find the magnitude of the electric field, as follows:

$$\begin{aligned}
\sum \vec{F} &= m\vec{a} \\
+eE &= m(+3.50 \times 10^{10}) \\
E &= \frac{m(3.50 \times 10^{10})}{e} \\
&= \frac{9.11 \times 10^{-31} \times 3.50 \times 10^{10}}{1.60 \times 10^{-19}} \\
E &= 0.199 \text{ N/C}
\end{aligned}$$

In conclusion, the electric field is 0.199 N/C directed to the south. Ans

Remarks

The above procedure has determined the magnitude and direction of electric field $\vec{E}$ separately. However, we can find both the magnitude and direction of the electric field at the same time by vector method, as the following:

The resultant force acting on the electron is the electric force, which is given by

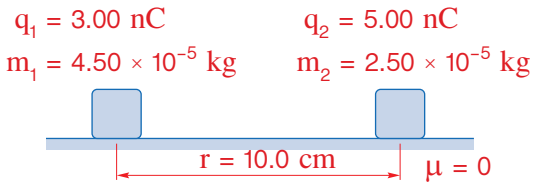
$$\sum \vec{F} = \vec{F}_E = q\vec{E} = -e\vec{E}$$

Using Newton's second law of motion, we get

$$\begin{aligned}
\sum \vec{F} &= m\vec{a} \\
-e\vec{E} &= m\vec{a} \\
\vec{E} &= -\frac{m\vec{a}}{e} \\
&= -\frac{9.11 \times 10^{-31} \times (+3.50 \times 10^{10})}{1.60 \times 10^{-19}} \\
&= -0.199 \text{ N/C} \\
\vec{E} &= 0.199 \text{ N/C directed to the south}
\end{aligned}$$

Problem 17.105

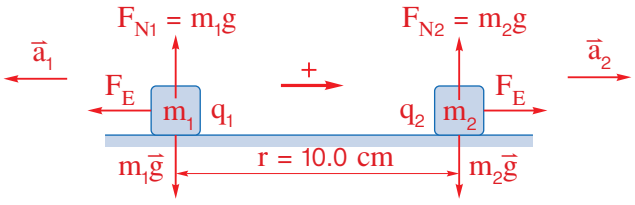
Two boxes are placed on a frictionless horizontal floor, 10.0 cm apart, as shown in Fig. (a), then release them to move. Find the acceleration of each box. Given the first box has a mass of 4.50×10^{-5} kg and a charge of 3.00 nC, the second box has a mass of 2.50×10^{-5} kg and a charge of 5.00 nC.



(a)

Solution

Since the charges on both boxes are positive, the electric force on each box is in the direction pushing the boxes away. When drawing the free-body diagrams of the boxes as in Fig. (b), the resultant force on each box is the electric force. Use the electric force as the resultant force, then find the acceleration of each box by applying Newton's second law of motion along the horizontal, as follows:



(b)

Set the direction to the right as positive.

The first box on the left:

The electric force acting on this box has magnitude $F_E = k_0 \dfrac{q_1 q_2}{r^2}$ directed to the left, which is the negative direction, so

$$\sum \vec{F} = m\vec{a}$$

$$-F_E = m_1\vec{a}_1$$

$$\vec{a}_1 = -\frac{F_E}{m_1}$$

$$= \frac{-k_0 \dfrac{q_1 q_2}{r^2}}{m_1}$$

$$= \frac{-k_0 q_1 q_2}{m_1 r^2}$$

$$= -\frac{8.99 \times 10^9 \times 3.00 \times 10^{-9} \times 5.00 \times 10^{-9}}{4.50 \times 10^{-5} \times 0.100^2}$$

$$= -0.300 \text{ m/s}^2$$

$\vec{a}_1$ = 0.300 m/s^2 directed to the left Ans

The second box on the right:

The electric force acting on the box is directed to the right in the positive direction, which the magnitude of this force is the same as that on the first box, so

$$\sum \vec{F} = m\vec{a}$$

$$+F_E = m_2\vec{a}_2$$

$$\vec{a}_2 = +\frac{F_E}{m_2}$$

$$= \frac{+k_0 \dfrac{q_1 q_2}{r^2}}{m_2}$$

$$= \frac{+k_0 q_1 q_2}{m_2 r^2}$$

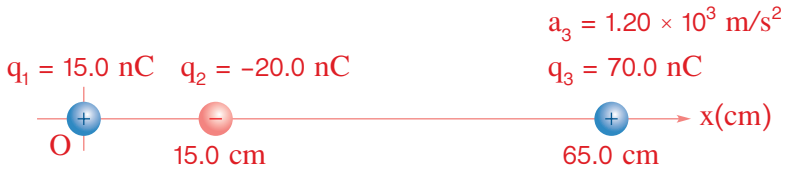
$$= +\frac{8.99 \times 10^9 \times 3.00 \times 10^{-9} \times 5.00 \times 10^{-9}}{2.50 \times 10^{-5} \times 0.100^2}$$

$$= +0.539 \text{ m/s}^2$$

$\vec{a}_2$ = 0.539 m/s^2 directed to the right Ans

Problem 17.106

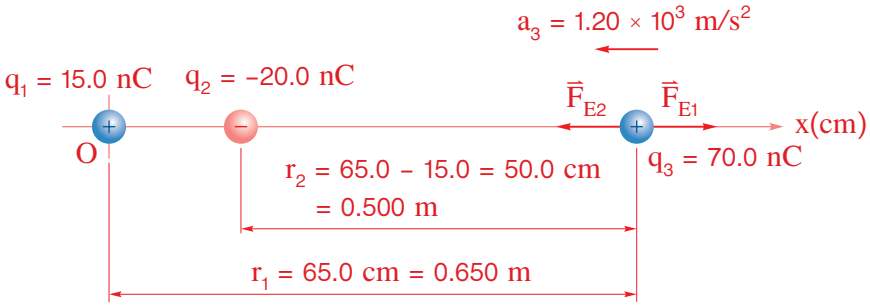
Two particles of charges $q_1 = 15.0$ nC and $q_2 = -20.0$ nC are fixed on the x-axis at the origin O and x = 15.0 cm, respectively, as shown in Fig. (a). Another particle has charge $q_3 = 70.0$ nC placed on the x-axis at x = 65.0 cm. When releasing this particle to move, its initial acceleration has a magnitude of 1.20×10^3 m/s^2. Find the mass of the third particle.



(a)

Solution

Charge q_1 produces electric force $\vec{F}_{E1}$ on charge q_3 directed along the positive x-axis, which is the direction pushing the particles away, because the two charges, q_1 and q_3, have the same sign, as shown in Fig. (b).



(b)

For charge q_2, it produces electric force $\vec{F}_{E2}$ on charge q_3 along the negative x-axis since the electric force is attractive between the charges, q_2 and q_3, of opposite signs.

From the forces acting on charge q_3, we find that the resultant force on charge q_3 is the vector sum of electric forces $\vec{F}_{E1}$ and $\vec{F}_{E2}$. The resultant force gives charge

q_3 an acceleration $\vec{a}$. Use Newton's second law of motion on charge q_3 to find its mass m, as follows:

$$\sum \vec{F} = m\vec{a}$$

$$\vec{F}_{E1} + \vec{F}_{E2} = m\vec{a} \quad \text{... (1)}$$

From Fig. (b), charges q_1 and q_3 are separated by distance r_1 = 65.0 cm = 0.650 m; charge q_2 is separated from charge q_3 by distance r_2 = 50.0 cm = 0.500 m. Thus, electric forces $\vec{F}_{E1}$ and $\vec{F}_{E2}$ are given by

$$\vec{F}_{E1} = k_0 \frac{q_1 q_3}{r_1^2} \vec{i} \quad \text{... (2)}$$

$$\vec{F}_{E2} = -k_0 \frac{|q_2| q_3}{r_2^2} \vec{i} \quad \text{... (3)}$$

Since $k_0 \frac{|q_2| q_3}{r_2^2}$ is greater than $k_0 \frac{q_1 q_3}{r_1^2}$, electric force $\vec{F}_{E2}$ has a greater magnitude than electric force $\vec{F}_{E1}$. So the resultant force is directed to the left in the direction of electric force $\vec{F}_{E2}$. It causes the acceleration directed to the left along the negative x–axis in the same direction as the resultant force, that is

$$\vec{a} = -1.20 \times 10^3 \vec{i} \text{ m/s}^2 \quad \text{... (4)}$$

Substitute $\vec{F}_{E1}, \vec{F}_{E2}$ and $\vec{a}$ from Eqs. (2) through (4) into Eq. (1) to find mass m of particle q_3, as follows:

$$m = \frac{\vec{F}_{E1} + \vec{F}_{E2}}{\vec{a}}$$

$$= \frac{k_0 \frac{q_1 q_3}{r_1^2} \vec{i} + \left(-k_0 \frac{|q_2| q_3}{r_2^2} \vec{i} \right)}{-1.20 \times 10^3 \vec{i}}$$

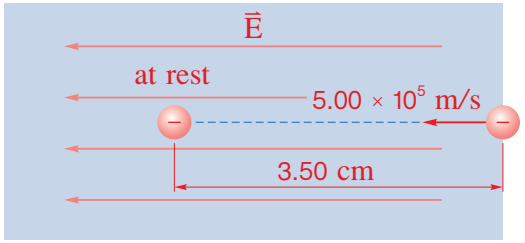
$$= \frac{k_0 q_3 \left(\frac{q_1}{r_1^2} - \frac{|q_2|}{r_2^2} \right) \vec{i}}{-1.20 \times 10^3 \vec{i}}$$

$$= \frac{8.99 \times 10^9 \times 70.0 \times 10^{-9} \left(\frac{15.0 \times 10^{-9}}{0.650^2} - \frac{\left|-20.0 \times 10^{-9}\right|}{0.500^2} \right) \vec{i}}{-1.20 \times 10^3 \, \vec{i}}$$

$$m = 2.33 \times 10^{-8} \text{ kg}$$ Ans

Problem 17.107

An electron moves to the left at a speed of 5.00×10^5 m/s entering a region of a uniform electric field. The direction of the electric field is parallel to the direction of the electron. The electric field causes the electron moving slower until it is at rest after traveling a distance of 3.50 cm, as shown in Fig. (a). Find the electric field.



(a)

Solution

When the electron enters the region of electric field $\vec{E}$, it experiences electric force $\vec{F}_E$. Since the electron moves slower, the electric force must have the opposite direction to the electron, which means directed to the right. The electron's charge is negative. Therefore, the electric force directed to the right shows that electric field $\vec{E}$ is directed to the left, the same direction as that of the electron, as shown in Fig. (a).

Electric field $\vec{E}$ produces an electric force on the electron, according to $\vec{F}_E = q\vec{E} = -e\vec{E}$. This force gives the electron an acceleration $\vec{a}$. Use Newton's second law of motion on the electron to find electric field $\vec{E}$:

$$\sum \vec{F} = m\vec{a}$$

$$-e\vec{E} = m\vec{a}$$

$$\vec{E} = -\frac{m\vec{a}}{e} \quad \text{... (1)}$$

An electron has mass $m = 9.11 \times 10^{-31}$ kg and charge magnitude $e = 1.60 \times 10^{-19}$ C. Using these data and Eq. (1), it is found that we require acceleration $\vec{a}$ to get the result of electric field $\vec{E}$. Therefore, we next determine acceleration $\vec{a}$ from the motion of the electron, as follows:

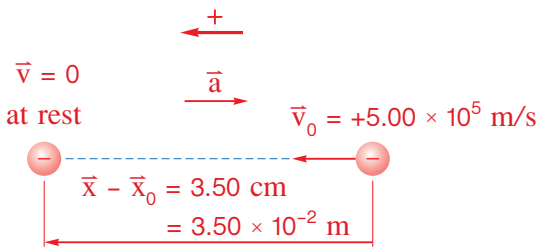
Since electric field $\vec{E}$ is uniform, electric force $\vec{F}_E = -e\vec{E}$ is constant, and it causes acceleration $\vec{a}$ constant. So we can find the acceleration of the electron by using the kinematics equations for one-dimensional motion with constant acceleration. Set the direction to the left as positive. The list of the quantities for the electron motion can be found in Fig. (b):

$$\vec{v}_0 = +5.00 \times 10^5 \text{ m/s}$$

$$\vec{v} = 0, \text{ (at rest at the final point)}$$

$$\vec{x} - \vec{x}_0 = +3.50 \text{ cm} = +3.50 \times 10^{-2} \text{ m}$$

$$\vec{a} = ?$$



(b)

From the above quantities, acceleration $\vec{a}$ can be determined by applying the following equation:

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{x} - \vec{x}_0)$$

$$0 = \vec{v}_0^2 + 2\vec{a}(\vec{x} - \vec{x}_0)$$

$$\vec{a} = \frac{-\vec{v}_0^2}{2(\vec{x} - \vec{x}_0)} \qquad \text{... (2)}$$

Substitute acceleration $\vec{a}$ from Eq. (2) into Eq. (1), then use the values for the above quantities to find electric field $\vec{E}$, as follows:

$$\vec{E} = -\frac{m}{e}\left(\frac{-\vec{v}_0^{\,2}}{2(\vec{x} - \vec{x}_0)}\right)$$

$$= \frac{m\vec{v}_0^{\,2}}{2e(\vec{x} - \vec{x}_0)}$$

$$= \frac{9.11 \times 10^{-31} \times (+5.00 \times 10^{5})^2}{2 \times 1.60 \times 10^{-19}(+3.50 \times 10^{-2})}$$

$$= +20.3 \text{ N/C}$$

$\vec{E}$ = 20.3 N/C in the same direction as the electron Ans

Problem 17.108

An electron travels into a uniform electric field at a speed of 1.50×10^6 m/s in the same direction as the electric field. Given the electric field has a magnitude of 250 N/C. When time elapses 2.00×10^{-8} s, find

1) the velocity of the electron
2) the distance traveled

Solution

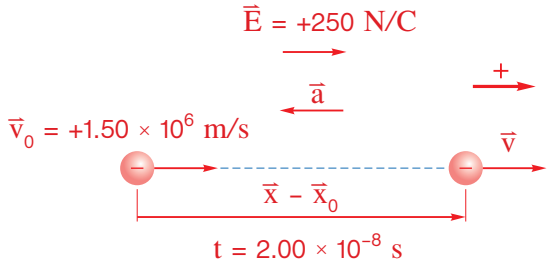
Uniform electric field $\vec{E}$ produces a constant electric force on the electron, according to equation $\vec{F}_E = q\vec{E} = -e\vec{E}$. The electric force is the resultant force acting on the electron. Therefore, the electron travels along a straight line with constant acceleration $\vec{a}$. The required quantities of the electron's motion are obtained by using the equations for straight-line motion with constant acceleration. However, we have to find constant acceleration $\vec{a}$ due to the electric force before, as follows:

Electric force $\vec{F}_E = -e\vec{E}$ gives the electron of mass m an acceleration $\vec{a}$. So we can find acceleration $\vec{a}$ by applying Newton's second law of motion:

$$\sum \vec{F} = m\vec{a}$$

$$-e\vec{E} = m\vec{a}$$

$$\vec{a} = -\frac{e\vec{E}}{m} \quad \text{... (1)}$$



1) From the figure, set the same direction as the velocity to be positive. The list of the motion quantities is

$$\vec{v}_0 = +1.50 \times 10^6 \text{ m/s}$$

$$t = 2.00 \times 10^{-8} \text{ s}$$

$$\vec{a} = -\frac{e\vec{E}}{m}, \quad \vec{E} = +250 \text{ N/C}$$

From the above quantities, the final velocity $\vec{v}$ of the electron is given by

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$= \vec{v}_0 - \frac{e\vec{E}}{m}t$$

$$= +1.50 \times 10^6 - \frac{1.60 \times 10^{-19} \times (+250)}{9.11 \times 10^{-31}} \times 2.00 \times 10^{-8}$$

$$\vec{v} = 6.22 \times 10^5 \text{ m/s}$$ Ans

2) The distance that the electron travels in time interval $t = 2.00 \times 10^{-8}$ s is the magnitude of displacement $\vec{x} - \vec{x}_0$, which is determined by using the following equation:

$$\vec{x} - \vec{x}_0 = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$= \vec{v}_0 t + \frac{1}{2}\left(-\frac{e\vec{E}}{m}\right)t^2$$

$$= +1.50 \times 10^6 \times 2.00 \times 10^{-8} - \frac{1}{2} \times \frac{1.60 \times 10^{-19} \times (+250)}{9.11 \times 10^{-31}} \times (2.00 \times 10^{-8})^2$$

$$= 2.12 \times 10^{-2} \text{ m}$$

$$\vec{x} - \vec{x}_0 = 2.12 \text{ cm}$$

Therefore, the electron travels a distance of 2.12 cm. Ans

Problem 17.109

An electron moves along the positive y-axis entering a region of uniform electric field $\vec{E} = 3.50 \times 10^3 \vec{j}$ N/C. If the electron travels a distance of 4.50 cm, then stops. Find

1) the acceleration of the electron
2) the initial velocity of the electron
3) the time interval from the electron entering the electric field until at rest

Solution

1) Electric field $\vec{E} = 3.50 \times 10^3 \vec{j}$ N/C produces electric force $\vec{F}_E = -e\vec{E}$ on the electron. This force is the resultant force that causes the electron to travel with constant acceleration $\vec{a}$. So we would determine acceleration $\vec{a}$ by applying Newton's second law of motion, as follows:

$$\sum \vec{F} = m\vec{a}$$

$$-e\vec{E} = m\vec{a}$$

$$\vec{a} = -\frac{e\vec{E}}{m}$$

$$= -\frac{1.60 \times 10^{-19} \times 3.50 \times 10^3 \vec{j}}{9.11 \times 10^{-31}}$$

$$\vec{a} = -6.15 \times 10^{14} \vec{j} \text{ m/s}^2$$ Ans

2) Since the electron travels along a straight line with constant acceleration $\vec{a}$ as found in 1), initial velocity $\vec{v}_0$ of the electron is obtained by applying constant-acceleration equations for straight-line motion. The list of the motion quantities is

$$\vec{a} = -6.15 \times 10^{14} \vec{j} \text{ m/s}^2$$

$$\vec{v} = 0$$

$$\vec{y} - \vec{y}_0 = +4.50\vec{j} \text{ cm} = +0.0450\vec{j} \text{ m}$$

Therefore, the initial velocity $\vec{v}_0$ is obtained by using the following equation:

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{y} - \vec{y}_0)$$

$$0 = \vec{v}_0^2 + 2(-6.15 \times 10^{14}\,\vec{j})(+0.0450\,\vec{j})$$

$$\vec{v}_0 = 7.44 \times 10^6\,\vec{j}\ \text{m/s} \qquad \text{Ans}$$

3) Time interval t is given by

$$\vec{y} - \vec{y}_0 = \vec{v}t - \frac{1}{2}\vec{a}t^2$$

$$= 0 - \frac{1}{2}\vec{a}t^2$$

$$t = \sqrt{-\frac{2(\vec{y} - \vec{y}_0)}{\vec{a}}}$$

$$= \sqrt{-\frac{2 \times (+0.0450\vec{j})}{-6.15 \times 10^{14}\vec{j}}}$$

$$t = 1.21 \times 10^{-8}\ \text{s} \qquad \text{Ans}$$

Problem 17.110

A proton travels with kinetic energy K entering a region of a uniform electric field along the electric field line. If the proton is at rest after moving distance s, find the electric field.

Solution

Since the proton enters the electric field with kinetic energy K, we would analyze the proton motion by work-energy theorem, as the following equation:

$$W_{net} = K_2 - K_1 \qquad \ldots (1)$$

The proton travels with kinetic energy K then stops, so $K_1 = K$ and $K_2 = 0$.

Net work W_{net} on the proton is caused by the electric force acting on the proton over distance s. Because the proton slows down, the electric force has the opposite

direction to the proton's motion and makes the net work negative. The electric force has magnitude $F_E = eE$, so $W_{net} = -F_E s = -eEs$.

Substitute all the above quantities into Eq. (1) to find the magnitude of the electric field, as follows:

$$-eEs = 0 - K$$

$$E = \frac{K}{es}$$

Since the charge on the proton is positive, the electric field and electric force have the same direction. In this case, the electric force is in the opposite direction to the motion, so the electric field has magnitude $\frac{K}{es}$ in the opposite direction to the motion. Ans

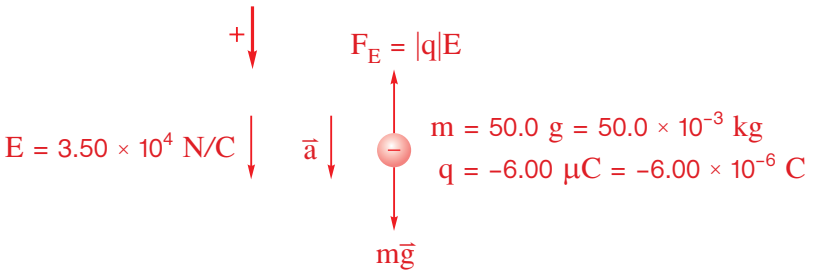
Problem 17.111

A small sphere of mass m = 50.0 g and charge q = −6.00 μC is thrown down at an initial velocity of 10.0 m/s in a region of a uniform electric field of 3.50×10^4 N/C in the downward direction. Find

1) the acceleration of the sphere
2) the velocity of the sphere after time elapses 3.00 s
3) the distance that the sphere travels in time interval 3.00 s

Solution

1) Start by drawing the free-body diagram of the sphere to consider the system of forces acting on it, then apply Newton's second law of motion to find the acceleration of the sphere.



From the figure, two forces are acting on the sphere: weight $m\vec{g}$ directed downward and electric force $\vec{F}_E$ directed upward. The electric force is directed upward since the charge on the sphere is negative, making the electric force directed upward opposite the downward direction of the electric field. Set the downward direction as positive. Apply Newton's second law of motion to find the acceleration of the sphere, as follows:

$$\sum \vec{F} = m\vec{a}$$

$$(+mg) + (-F_E) = m\vec{a}$$

$$\vec{a} = \frac{mg - |q|E}{m}, \quad F_E = |q|E$$

$$= g - \frac{|q|E}{m}$$

$$= 9.80 - \frac{\left|-6.00 \times 10^{-6}\right| \times 3.50 \times 10^{4}}{50.0 \times 10^{-3}}$$

$$= +5.60 \text{ m/s}^2$$

$$\vec{a} = 5.60 \text{ m/s}^2 \text{ directed downward}$$ Ans

2) From the result of 1), we find that the sphere moves down with constant acceleration $\vec{a} = +5.60$ m/s^2. So the velocity of the sphere after elapsed time 3.00 s is given by

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

where $\vec{v}_0 = +10.0$ m/s

$\vec{a} = +5.60$ m/s^2

$t = 3.00$ s

Therefore, $\vec{v} = (+10.0) + (+5.60) \times 3.00$

$$= +26.8 \text{ m/s}$$

$$\vec{v} = 26.8 \text{ m/s directed downward}$$ Ans

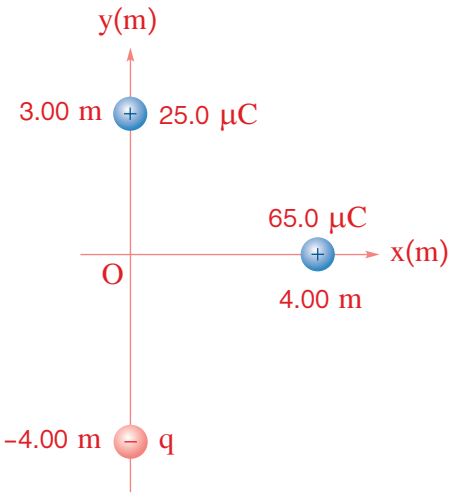
3) Since the sphere moves down without changing its direction, the required distance is the magnitude of the displacement within the same time interval, obtained by

$$\begin{aligned}\vec{y} - \vec{y}_0 &= \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \\ &= +10.0 \times 3.00 + \frac{1}{2} \times (+5.60) \times 3.00^2 \\ &= +55.2 \text{ m} \\ \vec{y} - \vec{y}_0 &= 55.2 \text{ m directed downward}\end{aligned}$$

In conclusion, within 3.00 s, the sphere travels down a distance of 55.2 m. Ans

Problem 17.112

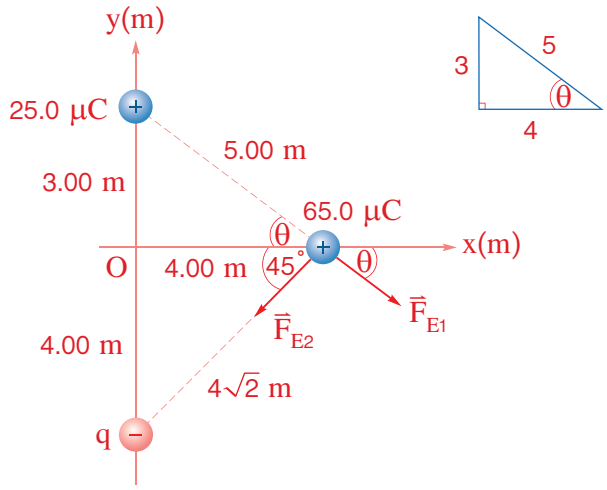
Charges 25.0 μC and q are fixed on the y-axis at y = 3.00 m and y = −4.00 m, respectively, as shown in Fig. (a). The third charge of 65.0 μC is placed on the x-axis at x = 4.00 m. This charge is free to move. When charge 65.0 μC is released to move, find charge q in which, at that time, charge 65.0 μC accelerates downward in the negative y-direction.



(a)

Solution

Concepts: the acceleration of charge 65.0 μC has the same direction as the resultant force acting on the charge. Therefore, start by considering the forces acting on the charge from the free-body diagram in Fig. (b).



(b)

The forces acting on charge 65.0 μC are two electric forces due to charges 25.0 μC and q. Force $\vec{F}_{E1}$ is the electric force due to charge 25.0 μC, which is repulsive downward to the right, making angle θ with the x-axis since charges 65.0 μC and 25.0 μC have the same sign. These two charges are separated by a distance of 5.00 m, so the magnitude of electric force $\vec{F}_{E1}$ is

$$F_{E1} = k_0 \frac{|q_1 q_2|}{r^2}$$

$$F_{E1} = k_0 \frac{25.0 \times 10^{-6} \times 65.0 \times 10^{-6}}{5.00^2} \quad \text{... (1)}$$

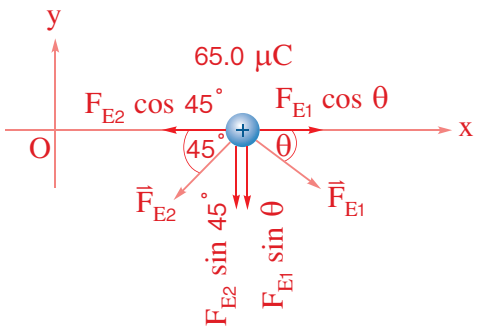
The second force is electric force $\vec{F}_{E2}$ due to charge q. Because the type of charge q is unknown, we cannot specify whether electric force $\vec{F}_{E2}$ is repulsive or attractive. However, the acceleration directed downward along the negative y-axis means the resultant force must also be directed downward along the negative y-axis. It makes us conclude that electric force $\vec{F}_{E2}$ is attractive downward to the left at an angle of 45° below the x-axis, as shown in Fig. (b). The reason is that when this force combines with electric force $\vec{F}_{E1}$, they can give the resultant force directed downward along the negative y-axis.

Electric force $\vec{F}_{E2}$ is attractive, so charges q and 65.0 μC have the opposite signs. Initially, we know that charge q is negative.

Charges q and 65.0 μC are separated by distance $4\sqrt{2}$ m, so the magnitude of electric field $\vec{F}_{E2}$ is

$$F_{E2} = k_0 \frac{|q_1 q_2|}{r^2}$$

$$F_{E2} = k_0 \frac{|q| \times 65.0 \times 10^{-6}}{(4\sqrt{2})^2} \quad \text{... (2)}$$



(c)

From Fig. (c), resolve electric forces $\vec{F}_{E1}$ and $\vec{F}_{E2}$ into the x- and y-components. We see that the resultant force is directed downward along the negative y-axis if the vector sum of the x-components is zero. The x-components have two forces in the opposite directions, therefore,

$$F_{E1} \cos\theta = F_{E2} \cos 45^\circ \quad \text{... (3)}$$

Substitute F_{E1} and F_{E2} from Eqs. (1) and (2) into Eq. (3) and use $\cos\theta = \frac{4}{5}$ from the right triangle of angle θ in Fig. (b) to find the magnitude of charge q, as follows:

$$k_0 \frac{25.0 \times 10^{-6} \times 65.0 \times 10^{-6}}{5.00^2} \times \frac{4}{5} = k_0 \frac{|q| \times 65.0 \times 10^{-6}}{(4\sqrt{2})^2} \cos 45^\circ$$

$$|q| = 36.2 \times 10^{-6} \text{ C}$$

$$|q| = 36.2 \text{ μC}$$

Since charge q is negative, it can be concluded that $q = -36.2\ \mu C$. Ans

Problem 17.113

A proton is held at rest in a uniform electric field $\vec{E} = (3.00 \times 10^{-7}\,\vec{i} - 2.50 \times 10^{-7}\,\vec{j})$ N/C at position $(x_0, y_0) = (1.00\ m, 2.00\ m)$, then released to move. Find the position of the proton when time elapses 2.00 s.

Solution

Since the proton is in the uniform electric field, it experiences a constant electric force, giving a constant acceleration. So our procedures begin by finding the proton's acceleration, then use it to seek the position of the proton by applying the constant-acceleration equations for straight-line motion, as follows:

A proton has mass $m = 1.67 \times 10^{-27}$ kg and charge $+e = 1.60 \times 10^{-19}$ C. When the proton is in electric field $\vec{E} = (3.00 \times 10^{-7}\,\vec{i} - 2.50 \times 10^{-7}\,\vec{j})$ N/C, it experiences electric force $\vec{F}_E = e\vec{E}$, giving the proton an acceleration $\vec{a}$, which is obtained by using Newton's second law of motion:

$$\sum \vec{F} = m\vec{a}$$

$$e\vec{E} = m\vec{a}$$

$$\vec{a} = \frac{e}{m}\vec{E}$$

$$= \frac{1.60 \times 10^{-19}}{1.67 \times 10^{-27}}(3.00 \times 10^{-7}\,\vec{i} - 2.50 \times 10^{-7}\,\vec{j})$$

$$\vec{a} = \frac{1.60 \times 10^{-19} \times 3.00 \times 10^{-7}}{1.67 \times 10^{-27}}\vec{i} - \frac{1.60 \times 10^{-19} \times 2.50 \times 10^{-7}}{1.67 \times 10^{-27}}\vec{j}$$

Given

$$\vec{a}_x = \frac{1.60 \times 10^{-19} \times 3.00 \times 10^{-7}}{1.67 \times 10^{-27}}\ m/s^2$$

$$\vec{a}_y = -\frac{1.60 \times 10^{-19} \times 2.50 \times 10^{-7}}{1.67 \times 10^{-27}}\ m/s^2$$

It can be seen that the proton travels with constant acceleration along both the x- and y-axes. Thus, apply the constant-acceleration equations for straight-line motion along the x- and y-axes separately to find the proton's position after elapsed time 2.00 s, as follows:

x-axis:

$$\vec{v}_{0x} = 0, \text{ start from rest}$$

$$\vec{x}_0 = 1.00 \text{ m}$$

$$\vec{a}_x = \frac{1.60 \times 10^{-19} \times 3.00 \times 10^{-7}}{1.67 \times 10^{-27}} \text{ m/s}^2$$

$$t = 2.00 \text{ s}$$

$$\vec{x} = \vec{x}_0 + v_{0x}t + \frac{1}{2}\vec{a}_x t^2$$

$$= 1.00 + 0(2.00) + \frac{1}{2}\left(\frac{1.60 \times 10^{-19} \times 3.00 \times 10^{-7}}{1.67 \times 10^{-27}}\right) \times 2.00^2$$

$$\vec{x} = 58.5 \text{ m}$$

y-axis:

$$\vec{v}_{0y} = 0, \text{ start from rest}$$

$$\vec{y}_0 = 2.00 \text{ m}$$

$$\vec{a}_y = -\frac{1.60 \times 10^{-19} \times 2.50 \times 10^{-7}}{1.67 \times 10^{-27}} \text{ m/s}^2$$

$$t = 2.00 \text{ s}$$

$$\vec{y} = \vec{y}_0 + v_{0y}t + \frac{1}{2}\vec{a}_y t^2$$

$$= 2.00 + 0(2.00) + \frac{1}{2}\left(-\frac{1.60 \times 10^{-19} \times 2.50 \times 10^{-7}}{1.67 \times 10^{-27}}\right) \times 2.00^2$$

$$\vec{y} = -45.9 \text{ m}$$

In conclusion, the proton is at position (58.5 m, -45.9 m) after elapsed time 2.00 s. Ans

Problem 17.114

A proton travels at initial velocity $(4.00 \times 10^5\,\vec{i} - 6.50 \times 10^5\,\vec{j})$ m/s entering uniform electric field $(2.20 \times 10^{-3}\,\vec{i} + 4.50 \times 10^{-3}\,\vec{j})$ N/C. Find the velocity of the proton after elapsed time 0.500 s.

Solution

Since the proton moves in the uniform electric field, it experiences a constant electric force, which gives it a constant acceleration. Begin by determining the proton's acceleration by applying Newton's second law of motion, then using the constant-acceleration equations for straight-line motion to find the proton's velocity after elapsed time 0.500 s. Here, we would do the calculation vectorially, as follows:

An proton has mass $m = 1.67 \times 10^{-27}$ kg and charge $+e = 1.60 \times 10^{-19}$ C. When the proton is in uniform electric field $\vec{E} = (2.20 \times 10^{-3}\,\vec{i} + 4.50 \times 10^{-3}\,\vec{j})$ N/C, it feels electric force $\vec{F}_E = e\vec{E}$. This force is the resultant force on the proton, which is used to find acceleration $\vec{a}$ by Newton's second law of motion, as the following:

$$\sum \vec{F} = m\vec{a}$$

$$e\vec{E} = m\vec{a}$$

$$\vec{a} = \frac{e}{m}\vec{E}$$

$$\vec{a} = \frac{1.60 \times 10^{-19}}{1.67 \times 10^{-27}}(2.20 \times 10^{-3}\,\vec{i} + 4.50 \times 10^{-3}\,\vec{j})\ \text{m/s}^2$$

Now acceleration $\vec{a}$ is known; use equation $\vec{v} = \vec{v}_0 + \vec{a}t$ to find velocity $\vec{v}$, where initial velocity $\vec{v}_0 = (4.00 \times 10^5\,\vec{i} - 6.50 \times 10^5\,\vec{j})$ m/s and time $t = 0.500$ s:

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

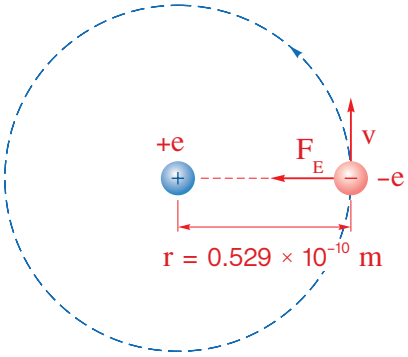
$$= (4.00 \times 10^5\,\vec{i} - 6.50 \times 10^5\,\vec{j}) + \frac{1.60 \times 10^{-19}}{1.67 \times 10^{-27}}(2.20 \times 10^{-3}\,\vec{i} + 4.50 \times 10^{-3}\,\vec{j}) \times 0.500$$

$$\vec{v} = (5.05 \times 10^5\,\vec{i} - 4.34 \times 10^5\,\vec{j})\ \text{m/s}$$ Ans

Problem 17.115

A hydrogen atom consists of an electron of mass 9.109×10^{-31} kg, moving along a circular path around its nucleus with a proton at an average radius of 0.529×10^{-10} m. Find the speed of the electron.

Solution



A proton has charge +e; an electron has charge –e. So, between the two particles, there are attractive electric forces. Considering the electron, as shown in the figure, this force is responsible for the centripetal force for the electron's circular motion. Apply Newton's second law of motion to the circular motion, where the electric force is the centripetal force, as follows:

Set the direction towards the center as positive. So the electric force on the electron can be written as

$$\vec{F}_E = +k_0 \frac{|q_1 q_2|}{r^2}$$

$$= k_0 \frac{|(+e)(-e)|}{r^2}$$

$$\vec{F}_E = k_0 \frac{e^2}{r^2}$$

Use the above electric force $\vec{F}_E$ as the centripetal force in the equation of the circular motion to find speed v of the electron:

$$\sum \vec{F}_c = m\frac{v^2}{r}$$

$$k_0\frac{e^2}{r^2} = m\frac{v^2}{r}$$

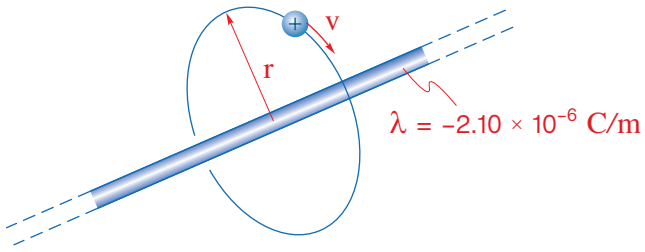
$$v = e\sqrt{\frac{k_0}{mr}}$$

$$= 1.60 \times 10^{-19}\sqrt{\frac{8.99 \times 10^9}{9.109 \times 10^{-31} \times 0.529 \times 10^{-10}}}$$

$$v = 2.19 \times 10^6 \text{ m/s}$$ Ans

Problem 17.116

A proton moves in a circular path of radius r around an infinite straight line with a uniformly distributed charge of linear charge density $\lambda = -2.10 \times 10^{-6}$ C/m, as shown in Fig, (a). Find the speed of the proton.



(a)

Solution

An infinite straight wire with a uniformly distributed charge of negative linear charge density λ produces an electric field surrounding the wire directed radially towards it. The electric field has magnitude $E = \frac{2k_0|\lambda|}{r}$, where r is the perpendicular distance from the wire.

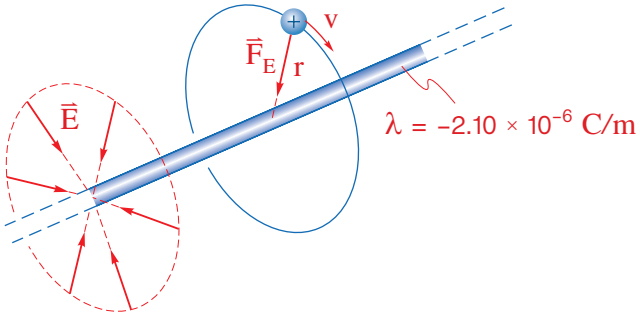
As described earlier, the proton, moving in a circular path with speed v around the wire, experiences electric force $\vec{F}_E$ due to electric field $\vec{E}$. This force is responsible for the centripetal force, as shown in Fig. (b). Use the electric force as

the centripetal force in the equation of Newton's second law for the circular motion, as follows:

$$\vec{F}_E = (+e)\vec{E}$$

$$= +e\left(\frac{2k_0|\lambda|}{r}\right)$$

$$\vec{F}_E = \frac{2k_0 e|\lambda|}{r}$$



(b)

Therefore,

$$\sum \vec{F}_c = \frac{mv^2}{r}$$

$$\frac{2k_0 e|\lambda|}{r} = \frac{mv^2}{r}$$

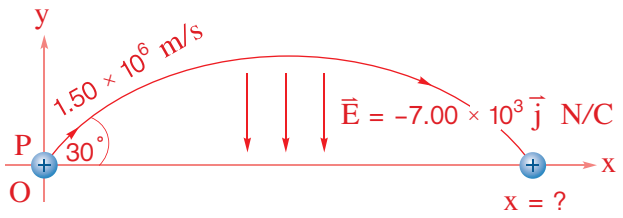
$$v = \sqrt{\frac{2k_0 e|\lambda|}{m}}$$

$$= \sqrt{\frac{2 \times 8.99 \times 10^9 \times 1.60 \times 10^{-19} \left|-2.10 \times 10^{-6}\right|}{1.67 \times 10^{-27}}}$$

$$v = 1.90 \times 10^6 \text{ m/s}$$ Ans

Problem 17.117

A proton is projected from the origin O with speed 1.50×10^6 m/s at a 30° angle with the positive x-axis, as shown in Fig. (a). If the proton moves in uniform electric field $\vec{E} = -7.00 \times 10^3\,\vec{j}$ N/C, at what position does the proton hit the x-axis?



(a)

Solution

A proton has a positive charge. When the proton moves in uniform electric field $\vec{E} = -7.00 \times 10^3\,\vec{j}$ N/C directed along the negative y-axis, it experiences a constant electric force directed along the negative y-axis, which is the same direction as the electric field. This force gives the proton a constant acceleration in the negative y-direction. So the proton's motion is similar to projectile motion, except that acceleration $\vec{g}$ due to gravity is replaced with acceleration $\vec{a}$ due to the electric force.

As said before, we begin by determining constant acceleration $\vec{a}$ due to the electric force acting on the proton, then analyzing the motion of proton by resolving it along the x- and y-axes. On the y-axis, the proton moves with constant acceleration $\vec{a}$, which is used to find the time traveled. Next, take the known time to find the proton's position from the motion along the x-axis with constant velocity, as follows:

The proton has charge +e, the electric force acting on the proton is therefore equal to $\vec{F}_E = +e\vec{E}$. This force is the resultant force that gives the proton constant acceleration $\vec{a}$. Find acceleration $\vec{a}$ by using Newton's second law of motion:

$$\sum \vec{F} = m\vec{a}$$

$$e\vec{E} = m\vec{a}$$

$$\vec{a} = \frac{e}{m}\vec{E}$$

$$= \frac{1.60 \times 10^{-19}}{1.67 \times 10^{-27}}(-7.00 \times 10^{3}\,\vec{j})$$

$$\vec{a} = -6.71 \times 10^{11}\,\vec{j}\ \text{m/s}^2$$

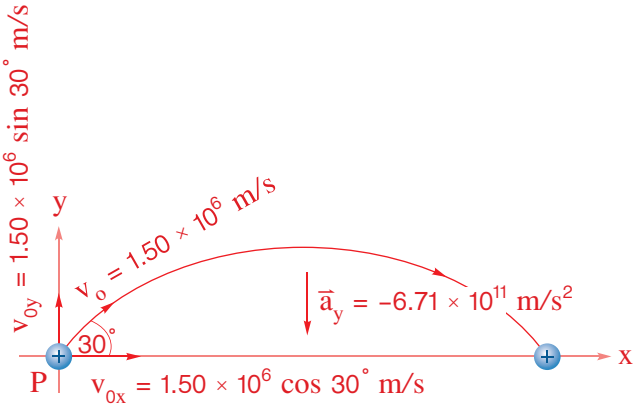
From the result of $\vec{a}$, we can see that the proton has a constant acceleration of 6.71×10^{11} m/s^2 directed along the negative y-axis. Along the x-axis, it has zero acceleration or constant velocity.

Resolve the proton's motion along the x- and y-axes, as shown in Fig. (b). From the figure, the list of motion quantities along the y-axis is

$$\vec{v}_{0y} = +1.50 \times 10^{6} \sin 30^{\circ}\ \text{m/s}$$

$$\vec{a}_{y} = -6.71 \times 10^{11}\ \text{m/s}^2$$

$$\vec{y} - \vec{y}_0 = 0, \text{ return to the original position on the y-axis}$$



(b)

Use the above quantities to find time t by the constant-acceleration equation, as follows:

$$\vec{y} - \vec{y}_0 = \vec{v}_{0y}t + \frac{1}{2}\vec{a}_y t^2$$

$$0 = (+1.50 \times 10^{6} \sin 30^{\circ})t + \frac{1}{2}(-6.71 \times 10^{11})t^2$$

$$t = \frac{2 \times 1.50 \times 10^6 \sin 30^\circ}{6.71 \times 10^{11}}$$

$$t = 2.24 \times 10^{-6}\ \text{s}$$

Along the x-axis, the proton moves at constant velocity $\vec{v}_{0x} = +1.50 \times 10^6 \cos 30^\circ$ m/s over time $t = 2.24 \times 10^{-6}$ s with initial position $\vec{x}_0 = 0$. So position $\vec{x}$ as the proton hits the x-axis is given by

$$\vec{x} - \vec{x}_0 = \vec{v}_{0x} t$$

$$\vec{x} = \vec{x}_0 + \vec{v}_{x0} t$$

$$= 0 + (+1.50 \times 10^6 \cos 30^\circ) \times 2.24 \times 10^{-6}$$

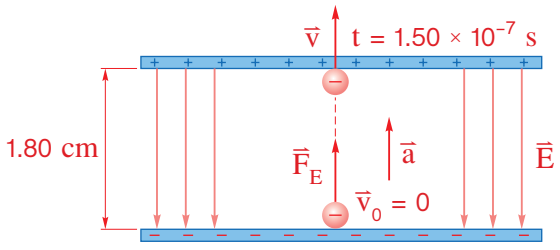
$$\vec{x} = 2.91\ \text{m}$$

In conclusion, the proton hits the x-axis at position (2.91 m, 0). Ans

Problem 17.118

Two flat sheets are placed parallel to each other, a distance 1.80 cm apart. The charges on the two plates have the opposite signs, producing a uniform electric field between the sheets. An electron is released from rest at the negatively charged sheet, then travels straight towards the positively charged sheet, and hits the sheet within a time interval of 1.50×10^{-7} s. Find the electric field and the electron's velocity as it hits the positively charged sheet.

Solution



Uniform electric field $\vec{E}$ between the sheets is directed from the positively charged sheet to the negatively charged sheet, as shown in the figure. The electric

field creates an electric force on the electron in the opposite direction to the electric field, that is, directed from the negatively charged sheet to the positively charged sheet. So the electric force causes the electron to move from the negatively charged sheet to the positively charged sheet.

Uniform electric field $\vec{E}$ between the sheets produces constant electric force $\vec{F}_E$ on the electron. This force gives the electron a constant acceleration $\vec{a}$. Therefore, apply Newton's second law of motion, where the resultant force is electric force $\vec{F}_E = q\vec{E} = -e\vec{E}$, as follows:

$$\begin{aligned} \sum \vec{F} &= m\vec{a} \\ -e\vec{E} &= m\vec{a} \\ \vec{E} &= -\frac{m}{e}\vec{a} \end{aligned} \qquad \text{... (1)}$$

Since the problem provides the data of the electron's motion, acceleration $\vec{a}$ is obtained by using constant-acceleration equations for straight-line motion. From the figure, set the upward direction from the negatively charged sheet to the positively charged sheet as positive. Therefore, the list of the motion quantities is

$$\begin{aligned} \vec{v}_0 &= 0, \text{ start from rest} \\ t &= 1.50 \times 10^{-7} \text{ s} \\ \vec{y} - \vec{y}_0 &= +1.80 \text{ cm} \\ &= +1.80 \times 10^{-2} \text{ m} \end{aligned}$$

Acceleration $\vec{a}$ is given by

$$\begin{aligned} \vec{y} - \vec{y}_0 &= \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \\ &= 0(t) + \frac{1}{2}\vec{a}t^2 \\ \vec{a} &= \frac{2(\vec{y} - \vec{y}_0)}{t^2} \end{aligned} \qquad \text{... (2)}$$

Substitute acceleration $\vec{a}$ from Eq. (2) into Eq. (1) to find electric field $\vec{E}$:

$$\vec{E} = -\frac{m}{e}\left(\frac{2(\vec{y} - \vec{y}_0)}{t^2}\right)$$

$$= -\frac{9.11 \times 10^{-31}}{1.60 \times 10^{-19}} \times \frac{2 \times (+1.80 \times 10^{-2})}{(1.50 \times 10^{-7})^2}$$

$$= -9.11 \text{ N/C}$$

$\vec{E}$ = 9.11 N/C directed outward from the positively charged sheet to the negatively charged sheet Ans

The speed of the electron as it attacks the positively charged sheet is the magnitude of the final velocity $\vec{v}$, which is given by

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$= 0 + \frac{2(y - \vec{y}_0)}{t^2}t$$

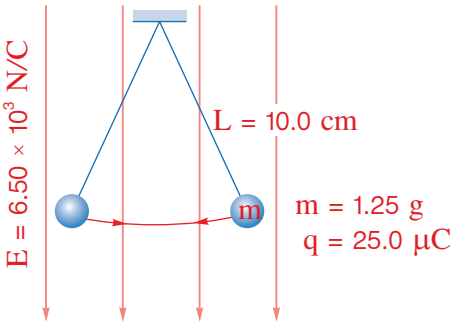
$$= \frac{2(y - \vec{y}_0)}{t}$$

$$= \frac{2 \times (+1.80 \times 10^{-2})}{1.50 \times 10^{-7}}$$

$\vec{v} = 2.40 \times 10^5$ m/s Ans

Problem 17.119

A small pendulum of mass 1.25 g and charge 25.0 μC is suspended by a light rope of length 10.0 cm and placed in a uniform electric field of magnitude 6.50×10^3 N/C directed downward, as shown in Fig. (a). When displacing the pendulum slightly from the vertical equilibrium position, then releasing it to oscillate as a simple pendulum, find the period of the pendulum.

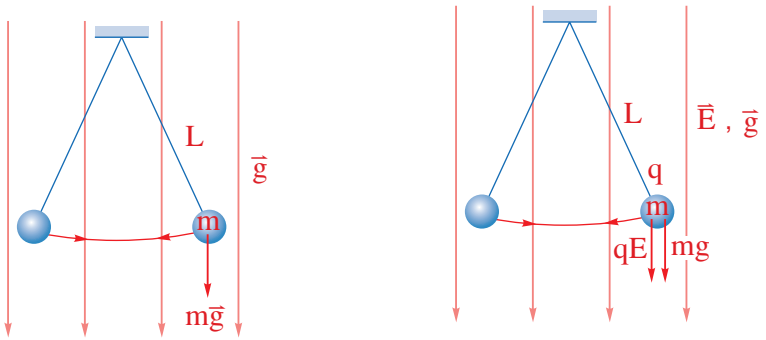


(a)

Solution

Let us go back to the topic of a simple pendulum. When a pendulum of mass m is suspended by a rope of length L and swings in gravitational field $\vec{g}$, as shown in Fig. (b), a component of weight $m\vec{g}$ causes the pendulum to oscillate, and period T of the oscillation is given by

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \ldots (1)$$



(b) (c)

In this problem, in addition to gravitational field $\vec{g}$, there is also electric field $\vec{E}$ directed downward, as shown in Fig. (c). So the force causing the pendulum to oscillate is the components of weight $m\vec{g}$ and electric force $\vec{F}_E = q\vec{E}$. Using the same steps deriving Eq. (1), we can write the expression for the period of the pendulum, as follows:

$$T = 2\pi\sqrt{\frac{L}{g + \frac{qE}{m}}} \qquad \ldots (2)$$

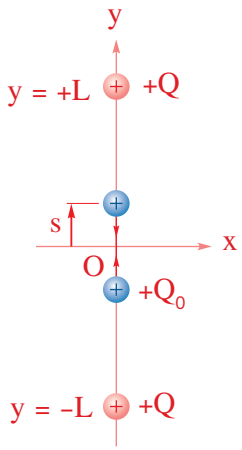
Substitute L = 0.100 m, m = 1.25×10^{-3} kg, g = 9.80 m/s^2, q = 25.0×10^{-6} C, and E = 6.50×10^3 N/C into Eq. (2) to find the period:

$$T = 2\pi\sqrt{\frac{0.100}{9.80 + \dfrac{25.0 \times 10^{-6} \times 6.50 \times 10^{3}}{1.25 \times 10^{-3}}}}$$

$$T = 0.168 \text{ s}$$ Ans

Problem 17.120

Two charges +Q are fixed on the y-axis at y = +L and y = -L, as shown in Fig. (a). Another charge $+Q_0$ is placed on the y-axis far away from the origin O by distance s that is much less than distance L. Show that when releasing charge $+Q_0$, the motion of the charge can be approximated as simple harmonic motion, and find the frequency of the oscillation.



(a)

Solution

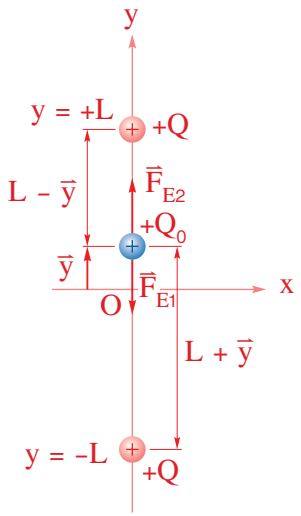
Concepts: an object oscillating as simple harmonic motion has a restoring force proportional to the negative of the displacement from its equilibrium position. If the oscillation occurs along the y-axis, that is, $\sum\vec{F} \propto -\vec{y}$ or $\sum\vec{F} = -k\vec{y}$, and the oscillation frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \ldots (1)$$

From the above concepts, we begin by finding the resultant force acting on charge $+Q_0$, then determine whether the resultant force is proportional to the negative of displacement $\vec{y}$ from the equilibrium position. If so, use Eq. (1) to calculate the frequency of the oscillation.

Figure (b) shows charge $+Q_0$ at any position $\vec{y}$. The forces acting on charge $+Q_0$ is two electric forces. The first force is $\vec{F}_{E1}$ due to charge $+Q$ at position $y = +L$, which is separated from charge $+Q_0$ by distance $L - \vec{y}$. This force is repulsive downward along the negative y–axis, so

$$\vec{F}_{E1} = -k_0 \frac{QQ_0}{(L - \vec{y})^2}$$



(b)

The second force is $\vec{F}_{E2}$ due to charge $+Q$ at $y = -L$. This force is repulsive upward along the positive y–axis in which charges $+Q$ and $+Q_0$ are separated by distance $L + \vec{y}$, therefore,

$$\vec{F}_{E2} = +k_0 \frac{QQ_0}{(L + \vec{y})^2}$$

The resultant force on charge $+Q_0$ is the vector sum of forces $\vec{F}_{E1}$ and $\vec{F}_{E2}$:

$$\sum \vec{F} = \vec{F}_{E1} + \vec{F}_{E2}$$

$$= -k_0 \frac{QQ_0}{(L - \vec{y})^2} + k_0 \frac{QQ_0}{(L + \vec{y})^2}$$

$$= k_0 QQ_0 \left(\frac{1}{(L + \vec{y})^2} - \frac{1}{(L - \vec{y})^2} \right)$$

$$= k_0 QQ_0 \left(\frac{(L - \vec{y})^2 - (L + \vec{y})^2}{(L + \vec{y})^2 (L - \vec{y})^2} \right)$$

$$\sum \vec{F} = k_0 QQ_0 \left(\frac{-4L\vec{y}}{(L + \vec{y})^2 (L - \vec{y})^2} \right) \quad \text{... (2)}$$

Since amplitude s << L, $\vec{y}$ << L. We can approximate that

$$L + \vec{y} \simeq L$$

$$L - \vec{y} \simeq L$$

Substituting the estimated value into Eq. (2), we get

$$\sum \vec{F} = k_0 QQ_0 \left(\frac{-4L\vec{y}}{L^2 \times L^2} \right)$$

$$\sum \vec{F} = \frac{-4k_0 QQ_0}{L^3} \vec{y} \quad \text{... (3)}$$

Because $\frac{4k_0 QQ_0}{L^3}$ is constant, Eq. (3) can be rewritten as

$$\sum \vec{F} = -k\vec{y} \ , \ k = \frac{4k_0 QQ_0}{L^3}$$

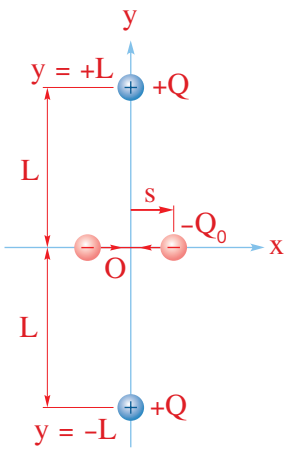
From the above result, it can be shown that when amplitude s << L, charge $+Q_0$ can be approximated to oscillate in a simple harmonic motion along the y-axis. So the frequency of the oscillation is given by Eq. (1), as follows:

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{4k_0 QQ_0}{mL^3}} \quad \text{Ans}$$

Problem 17.121

From the past problem, change charge $+Q_0$ to $-Q_0$, then release charge $-Q_0$ to move from an initial position on the x-axis away from the origin O by distance s, as shown in Fig. (a). Given distance s is much less than distance L. Show that charge $-Q_0$ oscillates in simple harmonic motion along the x-axis, and find its frequency.



(a)

Solution

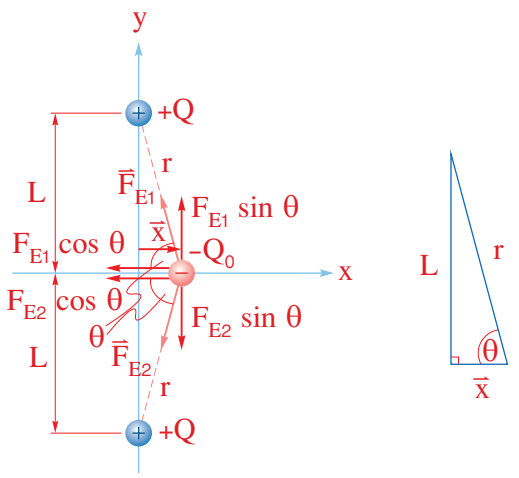
We will initially show that if amplitude s << L, charge $-Q_0$ oscillates in simple harmonic motion along the x-axis. To show this, find the resultant force on charge $-Q_0$, then prove that the resultant force is proportional to the negative of displacement $\vec{x}$, that is, $\sum \vec{F} \propto -\vec{x}$ or $\sum \vec{F} = -k\vec{y}$. Next, find the frequency of the oscillation by using the following equation:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \ldots (1)$$

Figure (b) shows charge $-Q_0$ at a position far from the origin O by displacement $\vec{x}$. At this position, charge $-Q_0$ is separated from both charges $+Q$ by the same distance r. Thus, electric forces $\vec{F}_{E1}$ and $\vec{F}_{E2}$ due to the two charges $+Q$ on charge $-Q_0$ have equal magnitude, as follows:

$$F_{E1} = F_{E2} = k_0 \frac{QQ_0}{r^2} \quad \text{... (2)}$$

where $$r^2 = L^2 + \vec{x}^2 \quad \text{... (3)}$$



(b)

Force $\vec{F}_{E1}$ and $\vec{F}_{E2}$ on charge $-Q_0$ are attractive, at an angle θ above and below the x-axis, respectively. When resolving these two forces into the horizontal and vertical components to find the resultant force on charge $-Q_0$, as shown in Fig. (b), we find that on the y-axis, there are two forces of equal magnitudes F_{E1} sin θ and F_{E2} sin θ ($F_{E1} = F_{E2}$) and opposite directions. Therefore, the vector sum of the y-components is zero, and the resultant force is only the vector sum of the x-components.

From Fig. (b), on the x-axis, there are two forces of equal magnitudes, F_{E1} cos θ and F_{E2} cos θ ($F_{E1} = F_{E2}$), directed towards the equilibrium position at the origin O along the negative x-axis. So the resultant force on charge $-Q_0$ is obtained by

$$\sum \vec{F} = -2F_{E1} \cos \theta \quad \text{... (4)}$$

Substitute F_{E1} from Eq. (2) into Eq. (4) and use $\cos \theta = \frac{\vec{x}}{r}$ from the right triangle of angle θ in Fig. (b), then use the value of r^2 from Eq. (3):

$$\sum \vec{F} = -2\left(k_0 \frac{QQ_0}{r^2}\right)\left(\frac{\vec{x}}{r}\right)$$

$$= -\frac{2k_0 QQ_0}{r^3} \vec{x}$$

$$\sum \vec{F} = -\frac{2k_0QQ_0}{(L^2 + \vec{x}^2)^{\frac{3}{2}}}\vec{x} \qquad \text{... (5)}$$

Since amplitude s << L, then $\vec{x}$ << L and we can estimate that $L^2 + \vec{x}^2 \simeq L^2$. Therefore, the resultant force in Eq. (5) can be approximated, as follows:

$$\sum \vec{F} = -\frac{2k_0QQ_0}{(L^2)^{\frac{3}{2}}}\vec{x}$$

$$= -\frac{2k_0QQ_0}{L^3}\vec{x}$$

Therefore, $\sum \vec{F} = -k\vec{x}$

where $k = \dfrac{2k_0QQ_0}{L^3}$

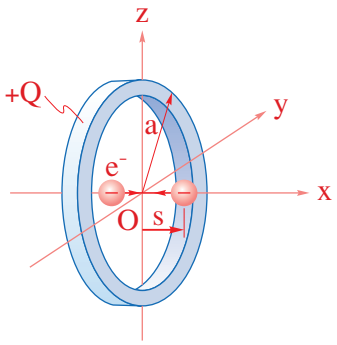
From the above result, when amplitude s << L, charge $-Q_0$ oscillates in simple harmonic motion along the x-axis, of which the frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{2k_0QQ_0}{mL^3}} \qquad \text{Ans}$$

Problem 17.122

A ring of radius a and uniformly distributed charge +Q is placed on the y-z plane, centered at the origin O, as shown in Fig. (a). An electron lies on the x-axis, a distance s from the ring's center, which is much less than radius a. Show that when the electron is released to move, it oscillates along the x-axis in simple harmonic motion and find the frequency of the oscillation.



(a)

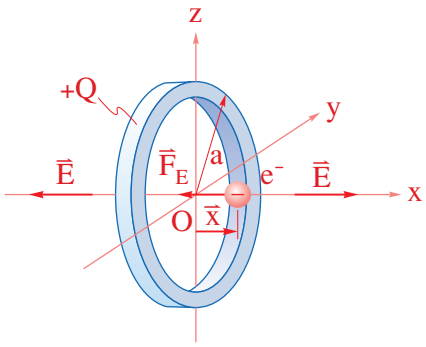
Solution

The procedure remains the same as the previous problem: find the resultant force on the electron, then show that the resultant force is proportional to the negative of the displacement according to the expression $\sum \vec{F} = -k\vec{x}$ to prove that the electron oscillates in simple harmonic motion along the x-axis. Finally, determine the frequency of the oscillation by using the following equation:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \text{... (1)}$$

The electric field due to charge +Q distributed uniformly on the ring of radius a is directed outward from the ring's center along the axis perpendicular to the plane of the ring and through the center, here the x-axis, as shown in Fig. (b). The magnitude of the electric field at position x is given by

$$E = \frac{k_0 |x| Q}{(x^2 + a^2)^{\frac{3}{2}}} \qquad \text{... (2)}$$



(b)

Consider an electron of charge –e placed on the x-axis away from the origin O by displacement $\vec{x}$, as shown in Fig. (b). Since the electron has a negative charge, electric field $\vec{E}$ produces electric force $\vec{F}_E$ on the electron directed towards the center of the ring. This force is the resultant force on the electron and obtained by

$$\sum \vec{F} = \vec{F}_E$$

$$= -e\vec{E}$$

$$= -e\left(\frac{k_0 \vec{x} Q}{(\vec{x}^2 + a^2)^{\frac{3}{2}}}\right)$$

$$\sum \vec{F} = -\frac{k_0 eQ\vec{x}}{(\vec{x}^2 + a^2)^{\frac{3}{2}}} \quad \text{... (3)}$$

Since distance s << a, displacement $\vec{x}$ << a. Therefore,

$$\vec{x}^2 + a^2 \simeq a^2$$

Equation (3) is reduced to

$$\sum \vec{F} = -\frac{k_0 eQ\vec{x}}{(a^2)^{\frac{3}{2}}}$$

$$\sum \vec{F} = -\frac{k_0 eQ}{a^3}\vec{x}$$

From the above expression, it can be seen that

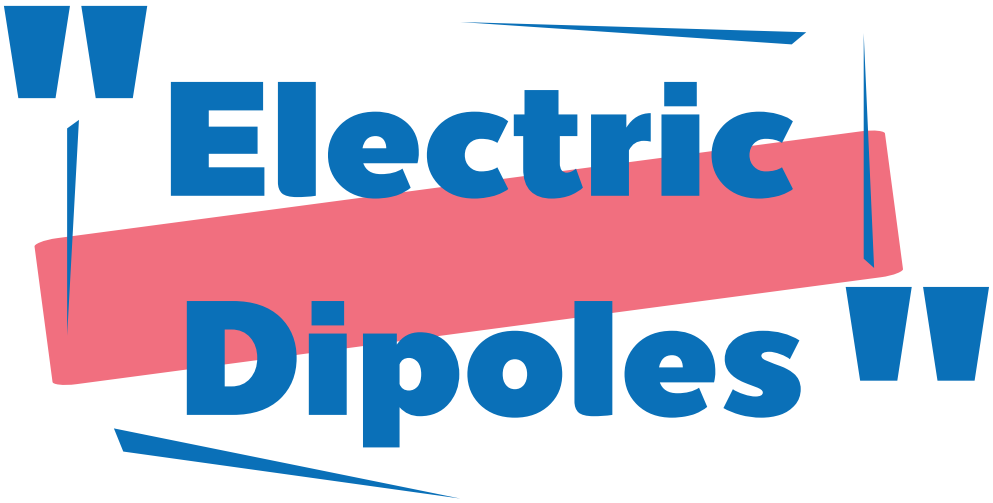
$$\sum \vec{F} = -k\vec{x}$$

where $$k = \frac{k_0 eQ}{a^3}$$

So the electron oscillates in simple harmonic motion along the x-axis, and its frequency is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k_0 eQ}{ma^3}}$$ Ans


"Electric
Dipoles"

Problem 17.123

A proton and an electron are placed 7.50 μm apart. Find the electric dipole moment.

Solution

A proton has a positive charge; an electron has a negative charge. Both particles have the same charge magnitude equal to $e = 1.60 \times 10^{-19}$ C. So they are an electric dipole.

Electric dipole moment $\vec{p}$ is directed from the negative charge to the positive charge. Thus, in this case, the electric dipole moment is directed from the electron to the proton and has a magnitude of

$$\begin{aligned} p &= qr_{(- \text{ to } +)} \\ &= ed \\ &= 1.60 \times 10^{-19} \times 7.50 \times 10^{-6} \\ p &= 1.20 \times 10^{-24} \text{ C·m} \end{aligned}$$

Ans

Problem 17.124

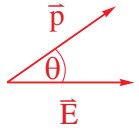
An electric dipole consists of charges ±6.75 nC placed 4.50 mm apart in a uniform electric field of magnitude 2.50×10^4 N/C. Calculate the magnitude of the torque on the electric dipole if the electric dipole moment has

1) the same direction as the electric field
2) the opposite direction to the electric field
3) direction perpendicular to the electric field

Solution

The torque magnitude is given by expression $\tau = pE \sin\theta$, where $p = qr_{(- \text{ to } +)} = qd$ and θ is the angle between electric dipole moment $\vec{p}$ and electric field $\vec{E}$, as

shown in Fig. (a). Use this expression to calculate the required magnitude of torque.



(a)

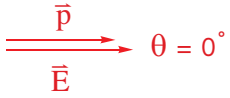
1) When the electric dipole moment has the same direction as the electric field, angle $\theta = 0^\circ$, as in Fig. (b). Therefore,

$$\tau = pE \sin\theta$$

$$= qdE \sin\theta$$

$$= 6.75 \times 10^{-9} \times 4.50 \times 10^{-3} \times 2.50 \times 10^{4} \sin 0^\circ$$

$$\tau = 0$$ Ans



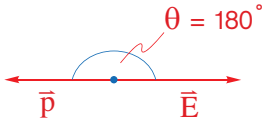
(b)

2) When the electric dipole moment has the opposite direction to the electric field, angle $\theta = 180^\circ$, as in Fig. (c). Therefore,

$$\tau = pE \sin\theta$$

$$= pE \sin 180^\circ$$

$$\tau = 0$$ Ans



(c)

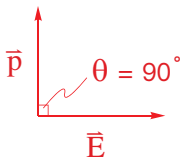
3) When the electric dipole moment is perpendicular to the electric field, angle $\theta = 90^\circ$, as in Fig. (d). Therefore,

$$\tau = pE \sin\theta$$

$$= qdE \sin\theta$$

$$= 6.75 \times 10^{-9} \times 4.50 \times 10^{-3} \times 2.50 \times 10^{4} \sin 90^{\circ}$$

$$\tau = 7.59 \times 10^{-7}\ \text{N·m}$$ Ans



(d)

Problem 17.125

What position of an electric dipole in uniform electric field $\vec{E}$ causes the torque on it equal to zero?

Solution

When an electric dipole of electric dipole moment $\vec{p}$ is in uniform electric field $\vec{E}$, the torque on the electric dipole is given by

$$\vec{\tau} = \vec{p} \times \vec{E}$$

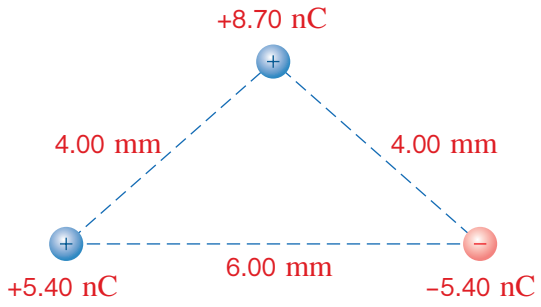
$$\tau = pE \sin\theta$$

where θ is the angle between $\vec{p}$ and $\vec{E}$

From the above expression, it can be seen that the torque is zero if $\sin\theta = 0$, which occurs when $\theta = 0^{\circ}$ or 180°. So the torque on the electric dipole is zero when the electric dipole moment is in the same direction or the opposite direction to the electric field. Ans

Problem 17.126

Charges +5.40 nC, −5.40 nC, and +8.70 nC are fixed at the corners of a triangle, as shown in Fig. (a). Find the magnitude of the electric force on charge +8.70 nC.



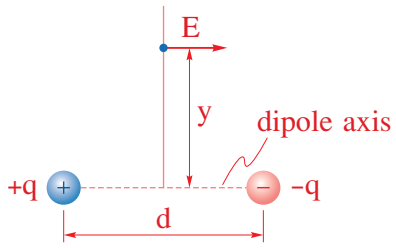
(a)

Solution

Charges ±5.40 nC form an electric dipole. So the electric force on charge +8.70 nC is due to the electric dipole.

In Fig. (b), for an electric dipole of charge ±q placed a distance d apart, the electric field due to the electric dipole at a point on the perpendicular bisector to the dipole's axis by distance y has magnitude

$$E = k_0 \frac{qd}{\left(\left(\frac{d}{2}\right)^2 + y^2\right)^{\frac{3}{2}}} \qquad \ldots (1)$$



(b)

When charge q_0 is placed at a field point with the electric field as in Eq. (1), the electric force acting on charge q_0 has magnitude

$$F_E = q_0 E$$

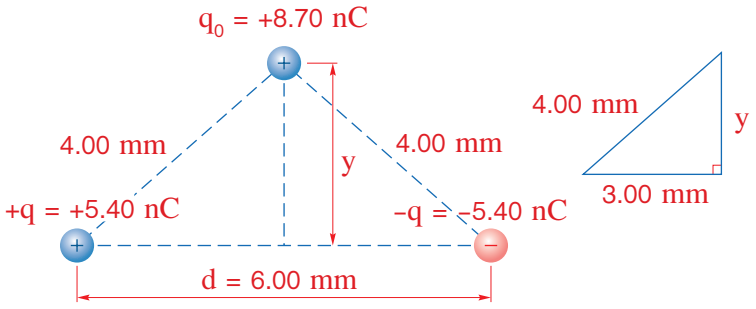
$$F_E = k_0 \frac{q q_0 d}{\left(\left(\frac{d}{2} \right)^2 + y^2 \right)^{\frac{3}{2}}} \qquad \text{... (2)}$$

Comparing Fig. (b) with Fig. (a), we get

$$q = 5.40 \text{ nC} = 5.40 \times 10^{-9} \text{ C}$$

$$q_0 = 8.70 \text{ nC} = 8.70 \times 10^{-9} \text{ C}$$

$$d = 6.00 \text{ mm} = 6.00 \times 10^{-3} \text{ m}$$



(c)

From the right triangle in Fig. (c), we get

$$y^2 = 4.00^2 - 3.00^2 \text{ mm}^2$$

$$= 7.00 \text{ mm}^2$$

$$y^2 = 7.00 \times 10^{-6} \text{ m}^2$$

Substituting the above quantities into Eq. (2), the magnitude of the electric force on charge +8.70 nC is

$$F_E = 8.99 \times 10^9 \times \frac{5.40 \times 10^{-9} \times 8.70 \times 10^{-9} \times 6.00 \times 10^{-3}}{\left((3.00 \times 10^{-3})^2 + 7.00 \times 10^{-6}\right)^{\frac{3}{2}}}$$

$$F_E = 3.96 \times 10^{-2}\ \text{N}$$ Ans

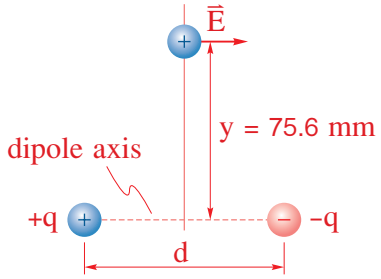
Problem 17.127

An electric dipole has an electric dipole moment of magnitude $p = 3.70 \times 10^{-25}$ C·m. Find the magnitude of an electric force on a proton, which is on the perpendicular bisector of the dipole's axis, far from the dipole center by distance 75.0 nm. Given distance 75.0 nm is much larger than the separation between the charges of the dipole.

Solution

For an electric dipole of charges ±q separated by distance d, as shown in the figure, an electric field at a point on the perpendicular bisector of the dipole axis, distance y from the dipole's center, has magnitude

$$E = k_0 \frac{qd}{\left(\left(\frac{d}{2}\right)^2 + y^2\right)^{\frac{3}{2}}} \quad \text{... (1)}$$



For distance y much greater than separation d between the charges, magnitude E of the electric field in Eq. (1) can be approximated as

$$E = k_0 \frac{qd}{y^3} \quad \text{... (2)}$$

Since p = qd,

$$E = k_0 \frac{p}{y^3} \qquad \text{... (3)}$$

When placing a proton in the electric field according to Eq. (3), the electric force acting on the proton has magnitude

$$F_E = eE$$

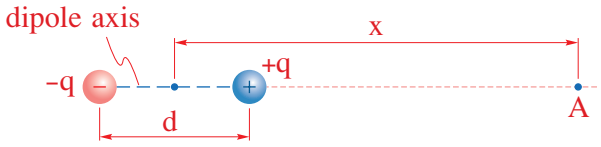
$$= e\left(k_0 \frac{p}{y^3}\right)$$

$$= 1.60 \times 10^{-19} \times 8.99 \times 10^9 \times \frac{3.70 \times 10^{-25}}{(75.0 \times 10^{-9})^3}$$

$$F_E = 1.26 \times 10^{-12}\ \text{N} \qquad \text{Ans}$$

Problem 17.128

An electric dipole of charges ±q are separated by distance d, as shown in Fig. (a). Find the electric field at point A along the dipole axis, away from the electric dipole's center by distance x. Given distance x is much greater than separation d between the charges (x >> d).

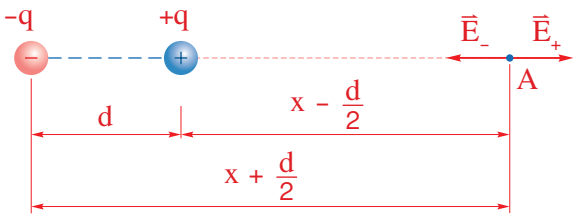


(a)

Solution

At point A, the net electric field is the vector sum of electric field $\vec{E}_+$ due to charge +q and electric field $\vec{E}_-$ due to charge −q, as shown in Fig. (b). Because point A is closer to charge +q than charge −q, electric field $\vec{E}_+$ has a greater magnitude than electric field $\vec{E}_-$. At point A, the net electric field is therefore directed to the right in the direction of electric field $\vec{E}_+$ and has magnitude

$$E = E_+ - E_- \qquad \text{... (1)}$$



(b)

From Fig. (b), point A is separated from charges +q and –q by distances $x - \frac{d}{2}$ and $x + \frac{d}{2}$, respectively. So magnitudes E_+ and E_- are

$$E_+ = k_0 \frac{q}{\left(x - \frac{d}{2}\right)^2}$$

$$E_- = k_0 \frac{q}{\left(x + \frac{d}{2}\right)^2}$$

Substituting E_+ and E_- into Eq. (1), we get

$$E = k_0 \frac{q}{\left(x - \frac{d}{2}\right)^2} - k_0 \frac{q}{\left(x + \frac{d}{2}\right)^2}$$

$$= k_0 q \left(\frac{1}{\left(x - \frac{d}{2}\right)^2} - \frac{1}{\left(x + \frac{d}{2}\right)^2} \right)$$

$$= k_0 q \frac{\left(x + \frac{d}{2}\right)^2 - \left(x - \frac{d}{2}\right)^2}{\left(x - \frac{d}{2}\right)^2 \left(x + \frac{d}{2}\right)^2}$$

$$E = k_0 q \left(\frac{2xd}{\left(x - \frac{d}{2}\right)^2 \left(x + \frac{d}{2}\right)^2} \right) \quad \text{... (2)}$$

Since x >> d, we can approximate that

$$x - \frac{d}{2} \simeq x$$

$$x + \frac{d}{2} \simeq x$$

Substitute the approximated value into Eq. (2):

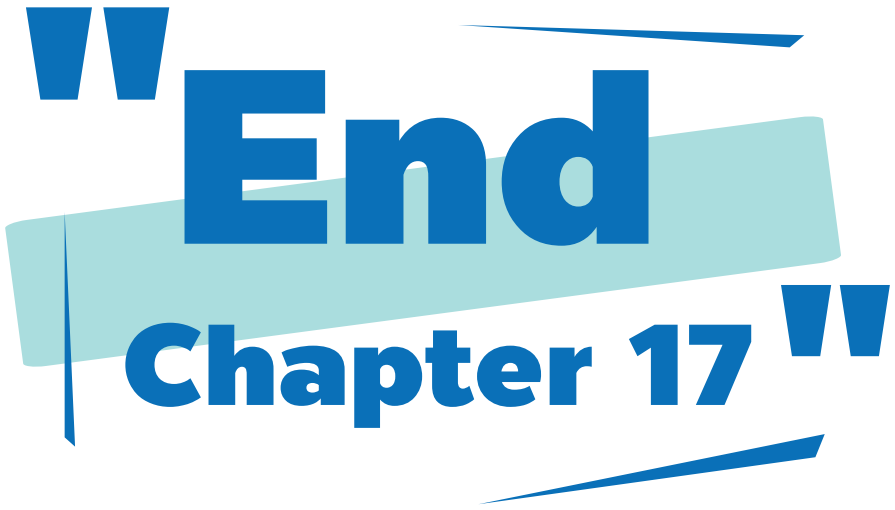
$$E = k_0 q \frac{2xd}{x^2 \times x^2}$$

$$E = k_0 \frac{2qd}{x^3} \quad \text{... (3)}$$

Since the electric dipole moment has magnitude p = qd, we can rewrite Eq. (3) as

$$E = k_0 \frac{2(qd)}{x^3} = k_0 \frac{2p}{x^3}$$

In conclusion, the electric field at point A has magnitude $k_0 \frac{2p}{x^3}$ directed to the right. Ans


"End
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